NPS-59BC74112A

NAVAL POSTGRADUATE SCHOOL
Monterey, California

STRESS ANALYSIS OF THERMOWELLS

by

John E. Brock

11 November 1974

Approved for public release; distribution unlimited.

NAVAL POSTGRADUATE SCHOOL
Monterey, California

Rear Admiral Isham Linder
Superintendent

J. R. Borsting
Provost

STRESS ANALYSIS OF THERMOWELLS

This monograph examines the mechanical-structural integrity of thermowells to sustain pressurization and excitations due to fluid flow. Suggested design criteria, which are shown to be conservative, are more inclusive than currently employed criteria, and in one important aspect, namely with respect to pressurization, are more liberal.

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NPS-59BC74112A	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) STRESS ANALYSIS OF THERMOWELLS		5. TYPE OF REPORT & PERIOD COVERED
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) John E. Brock		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Professor John E. Brock (Code 59Bc) Department of Mechanical Engineering Naval Postgraduate School		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE 11 November 1974
		13. NUMBER OF PAGES 107
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Thermowells
Cantilever vibration
Externally pressurized cylinders
Stress analysis
Tapered cantilevers

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This monograph examines the mechanical-structural integrity of thermowells to sustain pressurization and excitations due to fluid flow. Suggested design criteria, which are shown to be conservative, are more inclusive than currently employed criteria, and in one important aspect, namely with respect to pressurization, are more liberal.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
(Page 1) S/N 0102-014-6601 |

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

TABLE OF CONTENTS

A thermowell is a metallic product fitted into the wall of a pipe or vessel so as to permit introduction of a thermometer or thermocouple for the purpose of measuring the temperature of the contents. It is designed so as to maintain the integrity of the pressure boundary without introducing unacceptable measurement errors or time lags. This monograph summarizes the results of analytical studies made by the writer during the past two years of the mechanical/structural integrity of thermowells. It is obvious that much of this material is also applicable to other insertions such as sampling tubes, which, however, need not sustain the differential pressurization to which thermowells are subject.

An existing section (1) of the ASME Power Test Codes, based very largely upon analysis reported by J. W. Murdock (6), represents the consensus of the ASME Committee PB51 (on Thermowells). Designers have recently found it difficult to reconcile the strict requirements of this document with the practical necessity of providing thermowells for boiler feed discharge and main steam services. In the summer of 1972, Mr. J. E. Leary, Chief Control and Instrumentation Engineer of Bechtel Power Corporation, asked the writer to examine the structural integrity of thermowells and to compose recommendations for analysis of high pressure thermowells. A report (3) and a supplement (4) were produced shortly thereafter. A related study (5), by Professor T. M. Houlihan, examining the thermal performance of thermowells was also produced at this time; from this study it may be inferred that thermowell tip details which permit full assurance of structural integrity impose no problems of inaccurate temperature measurement or thermal time lag.

Subsequent to the production of the reports (3), (4), and (5), Mr. Leary asked the writer to solicit and collect comments from Mr. Murdock and members of the PB51 Committee. The writer is very pleased to acknowledge the participation and cooperation of the following persons:

Mr. J. W. Murdock, formerly of the Applied Physics Department, U. S. Naval Ship Engineering Center, Philadelphia, and presently a private consulting engineer.

Mr. L. A. Dodge, Bailey Meter Company

Mssrs. J. Archer and T. Reitz, Gilbert Associates

Mssrs. R. F. Abrahamsen and J. D. Fishburn, Combustion Engineering, Inc.

Mssrs. A. Lohmeier and A. J. Partington, Westinghouse Electric Corporation

Mr. W. N. Wright, TemTex Temperature Systems and Components

Mr. W. O. Hays, ASME (Secretary, PB51)

The most significant change recommended by the writer in his first report (3) was a drastic relaxation of the requirements with respect to simple pressurization. This matter is discussed in detail in Appendix A hereto. In the first report, in what seems to have proved to be a mistaken attempt at simplifying the presentation of the analysis, the writer considered internally pressurized hollow cylinders, a case to which most pertinent engineering literature has addressed itself. However, the point was established that with any material failure theory which is independent of the first scalar invariant of the stress tensor, the internally and externally pressured cases are equivalent if strains are limited to the order of magnitude of elastic strains. In the present Appendix A, the analysis is specifically directed to the externally pressurized case. However, precisely the same results are obtained as previously.

The comments generated in response to the writer's request dealt preponderately with the matter of pressurization and its consequences. While the need is acknowledged in the case of supercritical pressure installations to depart from the limitations imposed by strict application of the rules for externally pressurized vessels to be found in Section VIII of the ASME Boiler and Pressure Vessel Code, there is a reluctance actually to do so, and some responders feel that a test program is called for. However, the writer is absolutely certain in his own mind that internal pressurization tests would be absolutely useless. External pressurization tests might indeed prove useful as a basis for establishing rules even more liberal than the writer suggests (cf. Appendix A) should the need to do so ever arise. The writer's first report failed to cite the classic experiments of Prof. P. W. Bridgman (2) which should be adequately demonstrative of the ability of very thick wall, externally pressurized cylinders to retain pressure integrity even under extreme pressurizations. The comments of Mr. J. D. Fishburn conclude with the statement:

> "...In fact, Professor Brock's solution remains a conservative solution apart from the Code limitation of $S_m = (5/8)\bar{\sigma}$. This is for three reasons. Firstly, as stated by Professor Brock, 'for external pressurization...gross deformation is such as to modify the geometry advantageously,' secondly, a non-workhardening material is assumed, and, thirdly, there is considerable experimental evidence to show that the generalized Tresca condition for yield is of itself conservative.
>
> "For these reasons there should be no objections from the industry to this section of Professor Brock's analysis, particularly since the Nuclear Code specifies a value of $3S_m$ as the limiting stress range (Section III NM3222.2)."

Briefly, the writer, having studied all comments on the matter of pressurization and rules relating thereto, remains firm in his conviction that the liberalized criterion he recommends (cf. Section 10 of Appendix A) is still so conservative that it might be further liberalized if the need to do so should ever arise.

A second, and relatively unimportant pressure criterion relates to the thickness of tip closure for cantilevered thermowell. The requirement stated in Section II of Appendix A is simple and probably very conservative.

The third recommended criterion relates to the necessity of assuring that the mechanical excitation provided by the forces exerted on the thermowell by fluid flowing past it not be in resonance with a natural vibration mode of the thermowell structure. The analysis of this matter naturally divides itself into three aspects: (a) estimation of the exciting frequency, (b) estimation of the response (natural) frequency of thermowell vibration, and (c) consideration of how closely these may be permitted to approach one another.

The most troublesome and controversial of these subproblems is the first, which is discussed in detail in Appendix B hereto. For many years it was thought that the dimensionless parameter (called the Strouhal number) determining the frequency of vortex shedding for fluid flow past a fixed, rigid cylinder had a definite value (N_S = .21, appr.) for all flows faster than those characterized by Reynold's number (based on cylinder diameter) N_R = 800 (appr.) and Murdock's analysis (6) is based upon N_S = .21. However, more recent data, not available at the time of Murdock's study, indicate that the Strouhal number may become as great as N_S = .45 for large Reynold's numbers ($N_R = 6.2 \times 10^6$, appr.). This suggests that the frequency of excitation can become more than twice as great as previously contemplated. The writer's first report (3) employed this newer information in estimating the excitation frequency.

Comments received in this regard pointed out that there was lack of coherence in the vortex shedding which takes place for these higher values of Reynold's number and that, accordingly, the danger of resonant excitation is thereby diminished. There can be no argument with this contention but the question remains: to what extent is the probability of resonant excitation actually reduced? The experimental data invite a variety of interpretations and, in the writer's opinion, it is prudent to assume that coherent excitation of sufficient duration (perhaps as briefly as a fraction of a second at the frequencies involved) to cause damage can occur if excitation frequency based on $N_S = .45$ equals or exceeds the natural response frequency. Accordingly, the criteria recommended herein are based upon the possibility of coherent excitation with $N_S = .45$ in the appropriate range of Reynold's numbers. However, if these criteria are not met, it is suggested that the designer-engineer feel free to re-examine the matter, making use of whatever new information may at that time be available and making a special examination of the probability and consequences of coherent excitation. In correspondence with the PB51 Committee, the writer has suggested that the use of appropriate vibration test instrumentation applied to existing thermowell installations can possibly provide information which will help to assess the dangers of excitation in this regime. It should be made clear that this area is one in which persuasive information is indeed lacking and the writer's recommended criteria are intended to be definitely conservative.

The second of these three subproblems is that of estimating the natural response frequencies of thermowell vibration. This matter is the subject of Appendix C hereof, in which, throughout, it is assumed that we are dealing with a thermowell which is firmly attached to the pipe or vessel wall at one end (the root) and is free at the other end (the tip). Section 14 of Appendix C discusses a structural mode, not elsewhere discussed except in the supplement (4) to the writer's earlier report, which involves ovalization of the pipe or vessel and which is at relatively low frequency. An argument is offered for regarding this mode as of no practical significance, but it would certainly be desirable to have experimental evidence that this is the case.

The bulk of Appendix C relates to the estimation of the lowest mode of cantilever beam vibration. There are several complicating factors. The structure is nonuniform and is so short and stubby that rotatory inertia and shear deformation have a significant effect in reducing the response frequency as compared to what might be calculated by the use of so-called "elementary" theory. Furthermore, the pipe or vessel wall to which the thermowell is attached is itself flexible and possesses mass and this causes a further reduction in response frequency, compared to the elementary assumption of fixed root.

In Appendix C an attempt is made to take all these effects into account. A dynamic study employing the powerful new tool of the "finite element method" (FEM) is clearly the best practical way to perform the analysis and such an investigation is currently under way as a thesis study by LT. H. L. Crego, USN. Lacking the results of such a study, a "scrambling effort" is made in Appendix C to provide a reasonably accurate method of estimating natural frequency, accounting for non-uniformity of section and for the several non-elementary mechanisms which tend to depress the frequency. Briefly, the recommendation in Appendix C is that the frequency be calculated for the non-uniform cantilever by use of elementary theory (some curves presenting the results of such calculation are included, cf. Figure C-5 in Appendix C) and that the depressing mechanisms be accounted for (approximately) by use of a frequency reduction factor.

This brings us to the third of the subproblems listed above. In his original study, Murdock (6) made the requirement $r < 0.8$, where r = ratio of excitation frequency to natural response frequency, basing this recommendation upon a discussion he had on this subject with Professor J. P. den Hartog. Because of the uncertainties surrounding the calculation of these frequencies, particularly the effects of rotatory inertia, shear deflection, and foundation compliance and inertia, the writer feld strongly that the requirement $r < 0.8$ should be reduced to $r < 0.4$ and he so recommended in his original report (3). In the supplement thereto (4) the writer made a first attempt at including some of the depressing effects, and, as a result, proposed raising the limiting value of r to 0.65. In the present study, reported in Appendix C hereof, the completeness of the estimate of the depressing effects is believed to be much better. With the degree of uncertainty markedly reduced, it is reasonable to liberalize the requirement on the ratio r to its original value, namely, again we require that $r < 0.8$. However, it is very important to note that the denominator in the expression for r must adequately account for the depressing effects of rotatory inertia, shear deflection, and foundation compliance and inertia. An appropriate way of doing so is by use of the frequency reduction factor. Finite element studies, currently in progress, should permit refining the analysis.

The pressure criterion and the non-resonance criterion are the most important criteria. However, we are also concerned with the gross effect and the fatigue effect of bending. Although the bending moment is obviously greatest at the root, for a tapered section, the section modulus varies in such a way that the maximum bending stress may not occur at the root as was assumed by Murdock (6) and in the writer's earlier analysis (3), the latter, incidentally, being marred by an analytical error. In Appendix D the matter of maximum bending stress and maximum stress intensity is investigated. It is found that the previous assumption that the maximum occurs at the root is indeed true except for thermowells which are sharply tapered or strongly shielded or both. Appendix E similarly studies the fatigue effects; in the analysis of fatigue stresses a stress intensification factor of 6.0 is assumed. This value is quite conjectural. It is believed to be conservative. However the opinion of engineers who deal daily with stress intensification factors is definitely solicited on this choice. If a value different from 6.0 should be recommended by competent authority, the numerical factors in the formula in Appendix E should be proportionately modified.

Appendices D and E represent no changes in philosophy as compared to their first presentation in the writer's earlier report. However, the new presentations include the effect of partial shielding from the fluid stream, something not taken into consideration earlier, and they correct a simple algebraic error which was introduced earlier and which was "incorrectly corrected" in some subsequent correspondence with the PB51 Committee.

The bulk of the analysis is presented in Appendices A through G hereof. Appendices A through E have been referred to above. Appendix F is a Plan and Sequence of Calculations, showing all the recommended criteria. Appendix G presents a Numerical Example.

The writer joins others who may object that the present study, which is almost exclusively theoretical, fails to reflect operational experience. The most earnest solicitation of information dealing with failure, leakage, malperformance, etc., of thermowells attributable to mechanical/structural considerations turned up only one case, that of a thermowell which began leaking around at the root connection after many years of successful service; in this case, the difficulty seemed almost certainly attributable to a poorly executed attaching weld. Mr. J. E. Leary has expressed the hope that lack of reports of other cases may indicate a corresponding lack of operational difficulties; the writer's past experience does not lead to as sanguine a hope, only to the conclusion that a previously observed reluctance to report or to discuss or even to reveal failures of any kind of any equipment in any service extends also to thermowells. However, the one instance cited above permits the writer to discourse upon his very strong conviction that thermowells, as well as any other devices or appurtenances the installation of which involves penetrating the pressure boundary, must be attached by full penetration welds (or their "equivalent" whatever that may be in a particular situation) and that, furthermore, the "branch connection reinforcement rules" must be satisfied. See the discussion in Section 11 of Appendix A, hereof.

Acknowledgments

This work has been substantially supported by Bechtel Power Corporation by arrangement with Mr. J. E. Leary, Chief Control and Instrument Engineer. The original report (3) and supplement (4) were consulting reports to Bechtel which kindly permitted their dissemination to the PB51 Committee and to other interested persons. Bechtel Power Corporation has also supported a considerable share of the subsequent correspondence with commentators and the reworking into the present form. John F. Wax, Inc., has also supported some of this work; as a result of this and similar charities, this corporation is no longer active. The Naval Postgraduate School has supplied stenographic service, computer facilities, and publication effort; this may be regarded as in support of the writer's activities

as a member of the Mechanical Design Committee of the USAS B31 Code Group, and, until his recent resignation (for reasons of austerity), as a member of the Working Group on Piping for Section III of the ASME Boiler and Pressure Vessel Code.

General Bibliography

(Most of the Appendices have their own bibliographies; cf. General Table of Contents and individual Tables of Contents for the Appendices.)

1. ASME, Power Test Codes, Instruments and Apparatus, Part 3, Temperature Measurement, Paragraphs 8-19, incl., pp. 7-9.
2. Bridgman, P. W., Large Plastic Flow and Fracture, McGraw-Hill 1952, Chapter 8, pp. 142-163.
3. Brock, J. E., Stress Analysis of Thermowells, report to Bechtel Power Corporation, October 1972.
4. Brock, J. E., Supplement (to preceding listing).
5. Houlihan, T., Thermowell Calculations (Accuracy and Response), report to Bechtel Power Corporation, October 1972.
6. Murdock, J. W., Power Test Code Thermometer Wells, J. Eng. Power (ASME Trans.), October 1952, pp. 403-416.

APPENDIX A

ELASTIC-PLASTIC BEHAVIOR OF EXTERNALLY PRESSURIZED HOLLOW CIRCULAR CYLINDERS

TABLE OF CONTENTS

1. Basic Analysis

We consider the elastic-plastic behavior of an externally pressurized hollow circular cylinder of inner radius a and outer radius b. We take this to be a one-dimensional problem with principal stresses σ_r, σ_θ, σ_z varying only with r. (There is no axial or circumferential variation.) We assume an ideal elastic-plastic material which satisfies the usual equations of linear elasticity in the "elastic region" and a generalized Tresca condition (Guest's theory) in the "plastic region." We assume, in general, external pressurization P sufficient to cause plastic behavior for $a < r < c$ and elastic behavior for $c < r < b$.

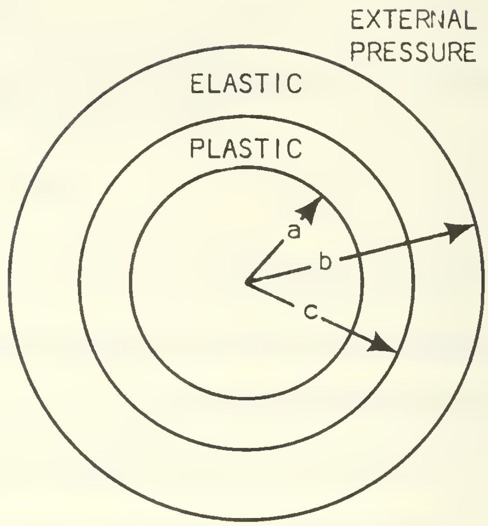


Fig. A.1 Section of partially plastic cylinder under external pressure.

The most fundamental relation which must be satisfied is that of radial equilibrium.

$$r(d\sigma_r/dr) = \sigma_\theta - \sigma_r \tag{1}$$

which is easily derived by use of a free body bounded by two normal planes having unit separation, two cylinders having radius r and r + dr respectively and two planes containing the axis and at a dehedral angle of $d\theta$ with each other.

We assume that the radial pressure is $q = -(\sigma_r)_{r=c}$ at the interface between the elastic and plastic regions. In the plastic region we assume (this can actually be shown) that $\sigma_\theta < \sigma_z < \sigma_r < 0$. Thus the Guest or Tresca condition is

$$\sigma_r - \sigma_\theta = \bar{\sigma} \tag{2}$$

where $\bar{\sigma}$ is the yield stress for the ideal elastic plastic material. The solution which satisfies (1) and (2) and the boundary condition $(\sigma_r)_{r=a} = 0$ is

$$\sigma_r = -\bar{\sigma}\ \ln(r/a);\ \sigma_\theta = -\bar{\sigma}[1 + \ln(r/a)] \tag{3a,b}$$

This gives

$$q = \bar{\sigma}\ln(c/a) \tag{4}$$

as the interface pressure and it also gives

$$(\sigma_\theta)_{r=c} = -q - \bar{\sigma} \tag{5}$$

Lamé's solution for the elastic region incorporates the usual elastic stress strain relations and the equilibrium relation (1). It gives

$$\sigma_r = [qc^2-Pb^2+(P-q)b^2c^2/r^2]/(b^2-c^2) \tag{6a}$$

$$\sigma_\theta = [qc^2-Pb^2-(P-q)b^2c^2/r^2]/(b^2-c^2) \tag{6b}$$

From (6a) it is easily verified that $(\sigma_r)_{r=c} = -q$ and $(\sigma_r)_{r=b} = -P$. From (6b) we find

$$(\sigma_\theta)_{r=c} = [q(b^2+c^2)-2Pb^2]/(b^2-c^2) \tag{7}$$

and equating this to the expression given by (5), one gets

$$P = q + \bar{\sigma}(b^2-c^2)/2b^2 \tag{8}$$

Using (4) one can also write

$$P = \bar{\sigma}[\ln(c/a) + (b^2-c^2)/2b^2] \tag{9}$$

and

$$P = q + (\bar{\sigma}/2)[1 - (a^2/b^2)\ln(2\bar{\sigma}/q)] \tag{10}$$

Equations (9) and (10) give P explicitely in terms of c and q respectively. However, given P, a more difficult evaluation is required to find c and/or q.

2. Elastic Pressure P_E and Ultimate Pressure P_U

If P is sufficiently small, c = a and q = 0. The Lamé solutions become

$$(b^2-a^2)\sigma_r = -Pb^2(1-a^2/r^2) \tag{11a}$$

$$(b^2-a^2)\sigma_\theta = -Pb^2(1+a^2/r^2) \tag{11b}$$

$$(b^2-a^2)\sigma_z = -Pb^2 \tag{11c}$$

The third of these is obtained by assuming an end of the cylinder is closed and that the external pressure also acts on the closure. Obviously the maximum principal stress difference is

$$\sigma_r - \sigma_\theta = 2Pa^2b^2/r^2(b^2-a^2) \tag{12}$$

which is a maximum at r = a, viz.

$$(\sigma_r-\sigma_\theta)_{r=a} = 2Pb^2/(b^2-a^2) \tag{13}$$

Thus, as P is increased, the Tresca condition is first encountered when

$$P = P_E = \bar{\sigma}(b^2-a^2)/2b^2 \tag{14}$$

The subscript E indicates <u>elastic</u> action for $P < P_E$.

On the other hand, for sufficiently high pressurization, the interface radius c assumes the value b and the interface pressure q assumes the value P, whence

$$P = P_U = \bar{\sigma}\ln(b/a) \tag{15}$$

3. Dimensional Changes

The subscript U denotes <u>ultimate</u>. However, in contradistinction with the usual case to which this word is applied, in the present case application of the "ultimate" load does not imply "plastic collapse" with "large" deformations. The reason for this is that the geometrical changes due to external pressurization are such as to increase the wall thickness.

Bridgman (3) has made a large strain analysis of externally pressurized cylinders and reference will later be made to his analysis. For the present, however, the following simple analysis will suffice.

Experiments by Bridgman and others and analysis by Bridgman indicates vanishingly small axial deformations even under pressures which result in gross change of diameters. Under plastic action, the volume remains constant. Thus, assuming initial radii a_1, b_1, and fully plastic action, new radii a_2, b_2 are obtained such that

$$P = \bar{\sigma}\ln(b_2/a_2);\ b_1^2 - a_1^2 = b_2^2 - a_2^2 \qquad (16a,b)$$

The first of these reflects fully plastic action (with no strain hardening) and the second reflects the volume constancy. We have assumed $P > P_U = \bar{\sigma}\ln(b_1/a_1)$, and, given P, $\bar{\sigma}$, a_1, and b_1 we wish to calculate a_2 and b_2. Using the notations

$$n = P/P_U > 1;\ \alpha = a_1/b_1 < 1 \qquad (17a,b)$$

we easily find

$$b_2 = b_1\sqrt{(1-\alpha^2)/(1-\alpha^{2n})};\ a_2 = b_2\alpha^n \qquad (18a,b)$$

For example, with $a_1 = .3$, $b_1 = 1.0$, a pressure three times as great as P_U gives d = .3, n = 3. Using equations (18) we find $b_2 = .9543$, $a_2 = .0258"$. The internal radius has been made quite small but equilibrium has been restored. If strain hardening occurs, the effective value of n is reduced and the distortion is not quite as great as indicated.

There is nothing, except limitations of pressurization facilities to restrict the value of n. For example, assuming $\bar{\sigma}$ = 65000 psi (accounting for heat treatment and some strain hardening) and an applied pressure P = 400000 psi, the value of P_U is 78260 psi so that n = 5.11. Then we calculate b_2 = .9539", a_1 = .0020". Clearly the final dimensions are not particularly sensitive to the value of n if $n \geq 2$. This calculation is consistent with experiments reported by Bridgman (3) except that Bridgman observed some cases for which the central cavity closed completely. Obviously it would take only a very, very small longitudinal contraction to cause this to happen.

The point of these recent remarks is to the effect that volume constancy acts to provide dimensional changes which restore equilibrium in the case of external pressurization whereas, in the case of internal pressurization, volume constancy gives a thinning of the wall which, in itself, acts to remove the situation even farther from equilibrium which can be restored, if at all, only by virtue of strain hardening. Thus catastrophic collapse <u>is</u> possible for internal pressure but it is not possible for external pressurization.

Accordingly, simply from the standpoint of maintaining pressure integrity, there is no theoretical limit to the external pressure which may be applied. However we are also concerned with maintaining a reasonable approximation to the original internal dimensions so that the thermocouple assembly may be withdrawn and replaced even when the pressure is applied.

For this reason, we now consider the dimensional changes which can be expected with $P = P_U$. The exterior surface is barely at the plastic stage so that we can use elastic formulas. Experiment indicates that we should take $\varepsilon_z = 0$, along with $\sigma_r = -P_U$, $\sigma_\theta = -\bar{\sigma} + \sigma_r = -\bar{\sigma}(1 + \ln b/a)$. We find

$$0 = E\varepsilon_z = \sigma_z - \nu(\sigma_\theta + \sigma_r); \quad \sigma_z = \nu(\sigma_\theta + \sigma_r)$$

$$E\varepsilon_\theta = \sigma_\theta - \nu(\sigma_r + \sigma_z) = \sigma_\theta - \nu\sigma_r - \nu^2(\sigma_\theta + \sigma_r)$$

$$= -(1-\nu^2)\bar{\sigma} + (1-\nu-2\nu^2)\sigma_r$$

$$= -(1-\nu^2)\bar{\sigma} - (1-\nu-2\nu^2)\bar{\sigma}\ln b/a \tag{19}$$

These values obtain at $r = b$ with $P = P_U$. Thus the radial deformation at $r = b$ is

$$\delta b = b\varepsilon_\theta = -\frac{b\bar{\sigma}}{E}[1-\nu^2+(1-\nu-2\nu^2)\ln b/a] \tag{20}$$

There is an ambiguity concerning the value of ν to employ. For P slightly less than P_U, $\nu=.3$ (approximately) whereas with P slightly greater than P_U, $\nu=.5$ but the elastic equations are not applicable. However, for our present purposes, the effect of ν does not alter the conclusion at which we shall arrive. We have

$$\varepsilon_\theta = \frac{\delta b}{b} = -(\bar{\sigma}/E)(.91+.52\ln b/a) \quad (\nu=.3)$$

$$= -.75\bar{\sigma}/E \quad (\nu=.5)$$

Taking, for example, a reasonable value of $\bar{\sigma}/E = 1.2 \times 10^{-3}$ and $b/a = 3$ we have

$$\frac{\delta b}{b} = -.0018 \quad (\nu=.3)$$

$$= -.0009 \quad (\nu=.5)$$

The larger value is probably more accurate and for our present purposes it is conservative.

Assuming volume consistency we have

$$(b+\delta b)^2 - (a+\delta a)^2 = b^2 - a^2 \tag{21}$$

so that, to first order

$$\delta a/a = (b/a^2)\delta b = (b/a)^2\delta b/b = -.0162$$

Thus, in a typical case, if a = .16", and b = .48", then δa = -.026". That is the interior radius shrinks by .026". This order of magnitude appears tolerable and our conclusion is that if the applied external pressure does not exceed P_U then the decrease in internal diameter will not adversely affect the ability to remove and replace thermocouple elements.

4. Repeated Pressurization, Elastic Shakedown, and the One-Cycle Shakedown Pressure P*.

We assume initial pressurization to a pressure P ($P_E < P < P_U$) so that there has been plastic behavior from r = a to r = c, (a < c < b). We presume however that subsequent removal of this pressure results in no additional plastic behavior. That is, the depressurization operation is purely elastic. Thus we can arrive at the final state of stress by superposing the Lamé stress system

$$\sigma_r = Pb^2(1-a^2/r^2)/(b^2-a^2) \tag{22a}$$

$$\sigma_\theta = Pb^2(1+a^2/r^2)/(b^2-a^2) \tag{22b}$$

upon the system given by equations 3 and 6. The maximum (equivalent or Tresca) stress state after depressurization occurs at r = a and is positive, i.e., at r = a we have

$$\sigma_r = 0,\ \sigma_\theta = -\bar{\sigma} + 2Pb^2/(b^2-a^2) \tag{23}$$

If the yield condition is not to be exceeded (this time in the opposite sense from originally), we must have

$$(\sigma_\theta)_{r=a} - (\sigma_r)_{r=a} < \bar{\sigma} \tag{24}$$

and this gives

$$Pb^2/(b^2-a^2) < \bar{\sigma};\ P < P^* = 2P_E = \bar{\sigma}(b^2-a^2)/b^2 \tag{25}$$

We will use the symbol $P^* = 2P_E$ and refer to the condition described above as "one cycle shakedown" since it assures that after the plastic yielding occurring on initial pressurization there can be no subsequent yielding.

Our previous analysis has led us to conclude that if $P \leqq P_U$ the deformations will be acceptable. Thus, we obviously wish to compare the pressures P_U and P^*. Equating these values, and using the notation $\alpha = a/b$, we immediately obtain the equation

$$1 + \ln\alpha = \alpha^2 \tag{26}$$

which has two roots, $\alpha = 1$, which is meaningless for our application, and

$$\alpha = 0.4503 \tag{27}$$

For $\alpha > 0.4503$, $P_U < P^*$ and if we required $P < P_U$, then one cycle shakedown is absolutely assured. For $\alpha < 0.4503$, one cycle shakedown requires limiting P to a maximum of $P^* < P_U$.

5. Basis of Recommended Criteria

We can represent our principal findings in a very compact form. Using the notation $\alpha = a/b$, we have

$$P^*/\bar{\sigma} = (1-\alpha^2), \quad P_U/\bar{\sigma} = -\ln(\alpha) \tag{28 a,b}$$

Our recommendation,

$$P \leqq \mathrm{Min}\{P_U, P^*\} = \mathrm{Min}\{-\ln(\alpha), (1-\alpha^2)\} \tag{29}$$

is indicated by the heavy line in Fig. A-2. This condition assures both

(a) not exceeding the ultimate pressure P_U, so that we are assured of tolerable dimension changes under pressurization, and

(b) one cycle shakedown so that all yielding occurs on initial pressurization and none on subsequent depressurization and repressurization.

This recommendation appears reasonable at this time. Since $\alpha < .4503$ for current designs of thermowells for high pressure applications, the criterion is conservative since surely there is no compelling reason to call for one cycle shakedown. Safe operation would also be assured if shakedown were to occur in two, three, or any other relatively small number

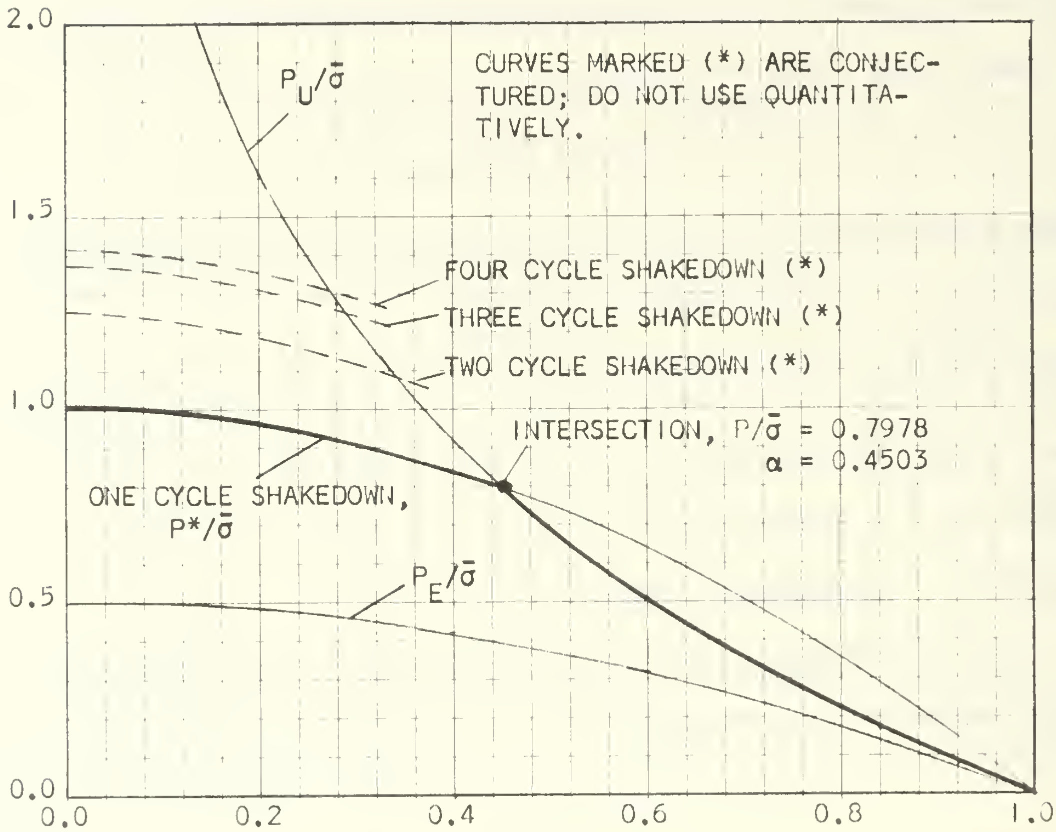


Fig. A-2 Summary of most significant pressure calculations.

of cycles. $P/\bar{\sigma}$ curves for such n-cycle shakedown have been conjecturally sketched in Fig. A-2 for n = 2,3, and 4; these curves should not be used quantitatively. The analytical difficulties involved do not presently warrant working out their actual shape. However, in the future, in any case where the presently recommended criterion should prove to be restrictive there would be ample reason to reconsider the matter so as to permit two or three cycle shakedown or so as to permit exceeding the pressure P_U.

6. Numerical example

So as to demonstrate the self consistency of the preceding analysis, it is desired to consider a practical case. We take $a/b = \alpha = 0.3$ and easily calculate $P_E = 0.4550\bar{\sigma}$, $P^* = 0.9100\bar{\sigma}$, and $P_U = 1.2040\bar{\sigma}$. We take

$P = P^*$ and let $\beta = c/b$. From equation 9, which becomes

$$1-\alpha^2 = \ln(\beta/\alpha) + (1-\beta^2)/2 \qquad (30)$$

we calculate $\beta = .51655$. This also gives $q/\bar{\sigma} = \ln\beta/\alpha = .5434$. Stress calculations are shown in Table A-1 and Figure A-3. The "final state" referred to is at the end of the first pressurization cycle, external pressure $P = .9/\bar{\sigma}$ having been applied once and then removed. After initial yielding, subsequent application and removal of $P = .9/\bar{\sigma}$ ceases stresses to vary between solid and dashed extremes.

TABLE A-1

Stress Calculations for Numerical Example

	PRESSURIZATION			REMOVAL		FINAL STATE		
Radius	$\sigma_\theta/\bar{\sigma}$	$\sigma_r/\bar{\sigma}$	$(\sigma_\theta-\sigma_r)/\bar{\sigma}$	$\sigma_\theta/\bar{\sigma}$	$\sigma_r/\bar{\sigma}$	$\sigma_\theta/\bar{\sigma}$	$\sigma_r/\bar{\sigma}$	$(\sigma_\theta-\sigma_r)/\bar{\sigma}$
.3b	-1.000	0	-1.000	2.000	0	1.000	0	1.000
.4b	-1.288	-.288	-1.000	1.562	.438	.274	.150	.124
.5b	-1.511	-.511	-1.000	1.360	.640	-.151	.129	-.280
c	-1.543	-.543	-1.000	1.337	.663	-.206	.120	-.326
.6b	-1.414	-.673	- .741	1.250	.750	-.164	.077	-.241
.7b	-1.316	-.771	- .545	1.184	.816	-.132	.045	-.177
.8b	-1.252	-.835	- .417	1.141	.859	-.111	.024	-.135
.9b	-1.208	-.879	- .329	1.111	.889	-.097	.010	-.107
b	-1.177	-.910	- .267	1.090	.910	-.087	0	-.087

7. Remarks About Experiments and Other Analyses.

In a previous analysis (4), the writer arrived at identical results and conclusions but the details may appear to be different than those given above. The reason is that in a misguided effort to make the presentation in terms of familiar material, the previous analysis (4) dealt with interior rather than external pressurization. Since the "failure law" (i.e., the Tresca condition) is independent of the hydrostatic stress state, i.e., is independent of the first scalar invariant of the stress tensor, internal pressurization is equivalent to external depressurization, and, with an

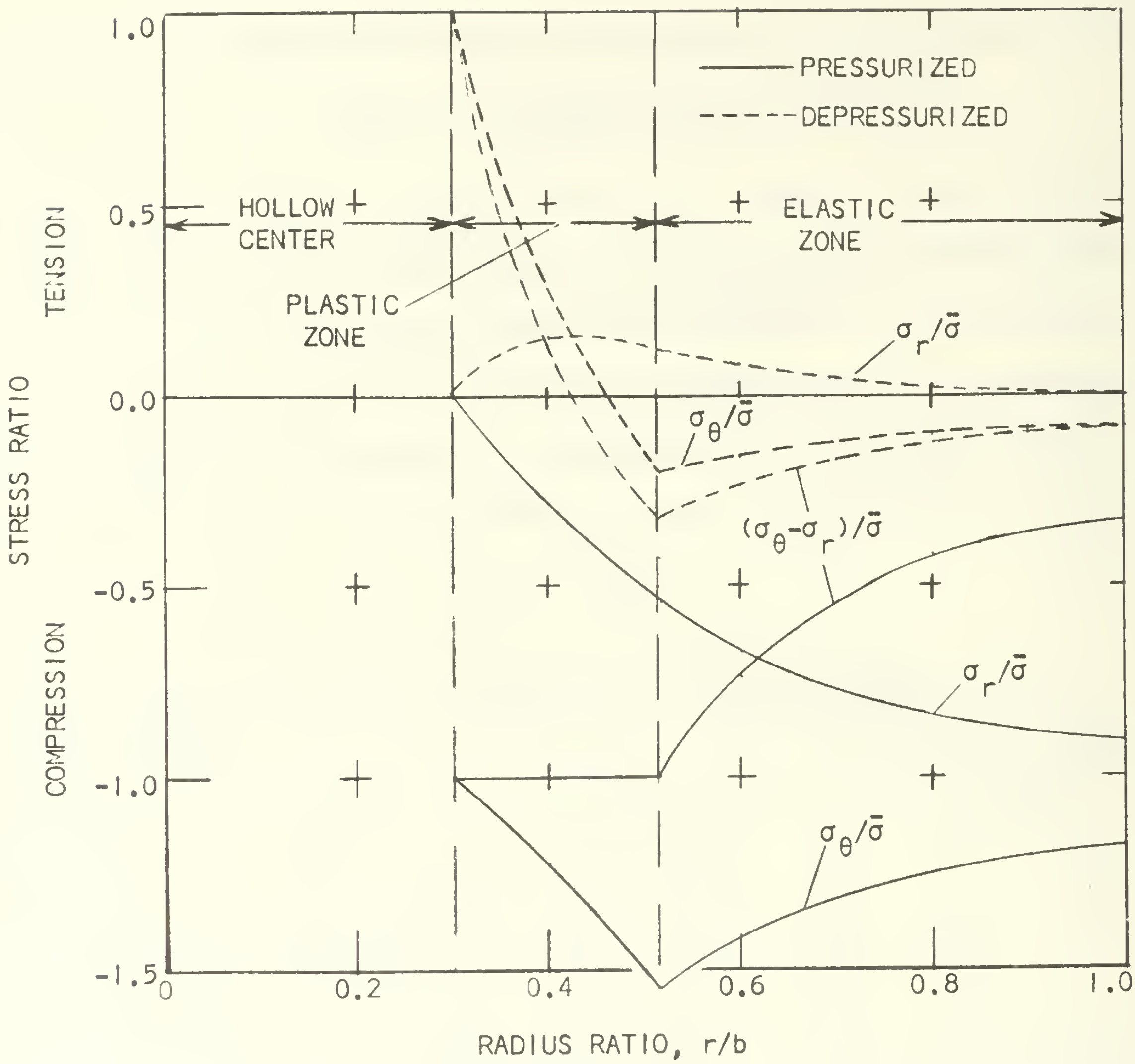


Fig. A-3 Stress distributions in numerical example.

overall change in algebraic sign, the latter is equivalent to <u>external pressurization</u>. The fact that the same conclusions are reached on the basis of the present analysis as were reached in (4) is sufficient evidence of the equivalence. However, instead of finding this viewpoint simplifying or illuminating, many readers of the earlier report were dismayed and confused. Accordingly, in the present analysis the actual, rather than an equivalent situation is treated.

The procedure here employs the simplest and most common plastic analysis to be found in the literature. The use of the Guest or Tresca condition is consistent with usage in the A.S.M.E. Boiler and Pressure Vessel Code. A readily available development (of the interior pressurization problem) is to be found in Timoshenko's popular textbook (12). The analysis is probably originally due to Nadai (9). However, it is not the only viewpoint. Bridgman (3) develops a generalized version and incorporates large-strain analysis; his development is essentially for fully plastic behavior. Hill et al. (6) consider modifications of the present analysis based on resolution of an undesirable discontinuity of axial strain at the elastic-plastic interface; their analysis predicts slightly less radial deformation than does the analysis given here for a given pressurization. A later work by Nadai (10) devotes three chapters to analysis of pressurized cylinders. A very recent work by Save and Massonnet (11) summarizes work to date and offers a large bibliography; they cite additional recent studies. The analysis summarized in (11) is the same as that given here.

Burst tests, such as described by Faupel (5) simply do not apply to the problem in which we are interested. However such tests have served to verify the general reliability of all the plastic analyses available; strain hardening is such as to mask differences.

Hill et al. (7) provide an analysis which indicates that if the Mises rather than the Tresca condition is used, P_U is increased by 15%, i.e., $P_U = (2/\sqrt{3})\bar{\sigma}\ln(b/a)$. Thus, use of the Tresca condition appears to be conservative.

Throughout the analysis and discussion to this point we have assumed axial symmetry. Specifically, we have not considered a mode of collapse in which the section becomes ovalized or goes out-of-round. The ASME "rules" for externally pressurized (thin wall) circular cylinders are based upon the predication of out-of-round deformation. Timoshenko and Gere (14) describe the genesis of the ASME procedure as combining a classical shell buckling analysis with what essentially amounts to the present formula (15), which, for thin wall, takes the form

$$P_U = \bar{\sigma} \ln[(1+t/2r_m)/(1-t/2r_m)] \simeq t\bar{\sigma}/r_m \tag{31}$$

as is given by the most elementary analysis. Ref (14) indicates also that the ASME rules of 1933 used an artificically low value of $\bar{\sigma}$ = 26000 psi; presumably modern versions of the ASME rules, which now provide curves for a number of different metals of engineering importance, reflect more realistic values of $\bar{\sigma}$. However, there do not appear to be <u>any</u> analyses in the literature which treat out-of-round buckling of externally pressurized cylinders having ratios $\alpha = a/b$ as large as those employed in thermowells for high pressure service.

Confining attention to thermowells for high pressure service, there seems to be no "engineering sense" in applying any criterion more restrictive than the one recommended in the preceding paragraphs. Externally pressurized cylinders having thick walls simply do not go out-of-round. Bridgman (3) conducted a number of tests on tubes of various steels having O.D. = .3125, I.D. = .0998. This corresponds to α = .32 which is approximately the value used in the examples in the present analysis and also corresponds to current industrial practice for high pressure installations. He subjected these tubes to as high as 412000 psi external pressure. In each case the tube simply decreased in diameter, while maintaining its length almost exactly without change. Equation (16b) was satisfied; that is, there was no volume change that could be detected. In one case, of a very soft steel under 412000 psi, the central cavity appeared to close up completely. However, there was no failure or loss of pressure integrity. Thus, for such thick wall cylinders, nonsymmetric distortion simply does not take place. The criterion recommended here (Equation 29) should assure that dimensional changes remain acceptably small and that shakedown to elastic conditions occurs promptly so that there is no danger of ratchetting or low-cycle fatigue.

8. Thin Wall Thermowells

The basic motivation behind this re-examination of design criteria for thermowells resides in the integrity of thermowells against very high external pressures, and the analysis thus far has been of such cases. However, design rules should cover all possible pressurizations, including, as an example, thermowells in exhaust gas ducts. However, it is not within the scope of the present study to consider such cases seriously or in detail. It seems reasonable to suppose that for case with "nominal" values of external pressure, the current interpretation of the Power Test Code, whatever that may be, should be considered applicable. In the appropriate part of this document (1) the maximum gage pressure is given by a formula $P = K_1 S$ where K_1 is a constant varying between 0.155 (for "large" thermocouple elements) to 0.412 (for "small" elements). If this criterion is satisfied for a thermowell having (roughly, say) the dimensions given in this document (1), there seems to be no reason to question the acceptability of the design on the basis of pressurization. For designs which must vary from the dimensions indicated in (1), it seems reasonable to attempt to apply the rules in the Unfired Pressure Vessel Code (2).

The only questions which appear to remain (on the question of wall thickness vs. pressure) are these. (1) For very low external pressurization, a certain degree of structural strength, possibly more than might be required by other criteria for thermowells, might be required to withstand the loads applied during shipment and installation or due to inadvertently applied mechanical loads during operation and maintenance, and (2) for "medium" pressure situations under what circumstances should one be concerned about true buckling in which ovalization or lobar deformation occurs.

We shall not concern ourselves further with (1) above. Let us turn attention to (2). The smallest value of D_o/t contemplated in the ASME Unfired Pressure Vessel Code (2)(Appendix V) is $D_o/t = 10$, and as was pointed out in (14), the criterion in this case is gross yielding with no change from the circular shape, using a substantial (>2) factor of safety. In the writer's opinion this is greatly overconservative for thermowells. However, let us not argue with it. We propose therefore that for $D_o/t \geq 10$, the Unfired Pressure Vessels be employed. The parameter $D_o/t = \frac{2}{1-\alpha}$ using the parameter $\alpha = a/b$ introduced earlier. Thus $D_o/t = 10$ corresponds to $\alpha = .8$. We believe that the strict application of the UFPV rules for $D_o/t < 10$ may be uneconomically conservative. Accordingly, we suggest that the entire gamut of pressure criteria be based as follows: (1) rules previously suggested for $0 \leqq \alpha \leqq .6$; (2) UFPV Code rules for $.8 \leqq \alpha \leqq 1.0$, (3) linear interpolation between.

9. Factor of Safety

Although the preceding discussion indicates no need for a "factor of safety" since catastrophic dimension change is not possible and even considerable overpressurization can result in nothing worse than squeezing down on the thermocouple element, nevertheless it is customary to provide for a factor of safety or its equivalant to account for inadvertent occasional overheating and/or overpressurization, the possibility of individual metallic specimens failing to possess the physical properties called for in the material purchase specification, deviations from design dimensions etc. The analysis here calls for knowing the yield stress $\bar{\sigma}$. The code allowable stress value, usually designated S_M, is readily available for all materials likely to be encountered and for all temperatures which might be employed. This stress value satisfies the inequality $S_M \leqq .625\bar{\sigma}$, so that $\bar{\sigma} \geqq 1.6\ S_M$. We propose using a "safety factor" of 1.6 simply by substituting the value S_M in place of the value $\bar{\sigma}$ in all our criteria.

10. Statement of Recommended Pressure Criteria.

Defining $\alpha = a/b$ = (inner radius)/(outer radius) = (inner diameter)/(outer diameter) = $(D_o-2t)/D_o = 1-2t/D_o$, we also have $D_o/t = 2/(1-\alpha)$. We also let S_M = code allowable "S-value" for the material and temperature and let P_{10} = allowable exterior pressure for $D_o/t = 10$, according to the UFPV Code. Then the recommended pressure criterion is:

(1) If $\alpha < .45$ (i.e., $D_o/t < 3.64$), $P \leqq (1-\alpha^2)S_M$

(2) If $.45 \leqq \alpha \leqq .6$ (i.e., $3.64 \leqq D_o/t \leqq 5$) $P \leqq -S_M \ln(\alpha)$

(3) If $.6 \leqq \alpha \leqq .8$ (i.e., $5 \leqq D_o/t \leqq 10$), $P = (4-5\alpha)(.51S_M) + (5\alpha-3)P_{10}$

(4) If $.8 \leqq \alpha \leqq 1$ (i.e., $D_o/t \geqq 10$), use the rules of Par. UG-28 of Division 1, Section VIII of the ASME Boiler and Pressure Vessel Code

Notes: (1) The dimensions employed in this calculation shall be those obtaining at the end of the design life of the thermowell. Accordingly, if corrosion may take place, an appropriate corrosion allowance should be added to exterior dimensions in order to arrive at manufacturing dimensions. (2) If dimensions are not constant along the length of the thermowell, the criterion given here must be satisfied at each cross section.

11. Analysis of Closure and Attachment.

Except as indicated in Note 2 of Section 10 (immediately above) the analysis here so far considers, in effect, an infinitely long thermowell with no influence due to restraint or other action of material at the ends, in the form of a pressure closure or attachment to pipe, vessel, or duct wall. The literature on plastic analysis of cylindrical shells including the influence of end or closure conditions indicates great analytic difficulties even in the case of thin shells. Accordingly, it seems to be out of the question to attempt to deal with this problem here in the case of thick shells.

We shall simply obtain a rough criterion for closure thickness and will remark on attachment details. For calculation purposes we deal with a circular plate of radius a and thickness t under lateral pressure P, and find that the maximum stress is

$$\sigma = kPa^2/t^2 \qquad (32)$$

The constant k depends on edge conditions (13). For simply supported edges, $k = 3(3+\nu)/8 = 1.24$ while for perfectly clamped edges $k = 0.75$. It is probably only slightly conservative to take $k = 1$. If we require $\sigma \leqq S_M$, we calculate

$$t = a\sqrt{P/S_M} \qquad (33)$$

A typical calculation gives $t = .123\sqrt{4750/7200} = .107"$. Analysis of the thermal transient behavior of the thermowell-thermocouple assembly indicates that tip thickness is not a significantly limiting factor (8). Accordingly, in order to provide safety against mechanical damage during shipment and installation it seems reasonable to ask for a greater tip thickness. The criterion

$$t_{AVG} \leqq 2a\sqrt{P/S_M} \qquad (34)$$

appears to be quite conservative and it also accommodates cases where closure thickness is not constant, such as a thermowell the interior cavity of which is formed by a twist drill.

There is simply no feasible way of investigating shakedown in the neighborhood of the tip (closure) or of the root (attachment to pipe wall) of a thermowell. The 1.6 "safety factor" previously introduced, together with the requirement of adequately thick closure and (see below) strong attachment details, should, however, assure shakedown immediately or very early in the operating life.

Practice differs with regard to attachment details. We will here discuss only the case of installations intended for high pressures. Some fabricator specifications appear to call for only sufficient thread engagement (in the case of threaded connections) or weld metal to assure that the thermowell assembly is not projected radially outward. In the writer's opinion this represents gross under-design. The only case of high pressure thermowell failure of which the writer has knowledge seem unquestionably to be associated with failure of the attachment weld (after satisfactory operation for a number of years, incidentally). When one considers all possible ways in which a thermowell could "fail" in a catastrophic or seriously disabling way, it seems clear that in any such case a marginally adequate attachment detail can not be other than a contributing factor.

In seventeen years association with the Mechanical Design Committee (of the B31 Code "family") the writer has consistently argued that the pressure carrying integrity of a pipe or header or run or vessel or whatever is compromised by any removal of material from the walls, whether this be for the purpose of making a branch connection, in which case reinforcement rules apply, or for any other purpose, in which case no rules seem to be called for. The hole which is made to insert a radiographic pellet for weld inspection should require no less attention than does a branch connection hole of the same size. The same is surely true for the hole made to accommodate a thermowell installation. If the pipe into which the installation is made has substantial excess thickness over that required by the applicable pipe wall thickness formula, then perhaps a case can be made for less than full penetration welds. Otherwise, it is absolutely clear to this writer that full penetration welds are called for, at the very least, and that, perhaps, additional reinforcement may be required. This should be determined by a strict application of the rules for reinforcement of branch connections, noting, however, that the branch itself is, in this case, not internally pressurized.

Accordingly, as developed in this subsection of this Appendix A, two additional criteria hereby recommended are: (1) Average thickness of end closure not less than $2a\sqrt{P/S_M}$, and minimum thickness not less than $a\sqrt{P/S_M}$, and (2) strict application of branch connection reinforcement rules to the detail of attaching thermowell to pipe wall, with full penetration welds in all cases.

<u>12. Bibliography for Appendix A</u>

A1. ASME, Power Test Codes, Instruments and Apparatus, Part 3 Temperature Measurement, Paragraphs 8-19 incl., pp. 7-9.

A2. ASME, Boiler and Pressure Vessel Code, Section VIII, Pressure Vessels, Division 1, Paragraph UG-28 and Appendix V.

A3. Bridgman, P. W., Large Plastic Flow and Fracture, McGraw-Hill 1952, Chapter 8, pp. 142-163.

A4. Brock, J. E., Stress Analysis of Thermowells, consulting report for Bechtel Power Corporation, October 1972.

A5. Faupel, J. H., Yield and Bursting Characterisitcs of Heavy-wall Cylinders, Trans. ASME, Vol. 78, 1956, pp. 1031 et seq.

A6. Hill, R., Lee, E. H., and Tupper, S. J., The theory of combined plastic and elastic deformation with particular reference to a thick tube under internal pressure, Proc. Roy. Soc. Lond., A, 191, pp. 278-303, 1947.

A7. Hill, R. Lee, E. H., and Tupper, S. J., Plastic flow in a closed tube with internal pressure, Proc. 1st U. S. Nat. Cong. Appl. Mech., Chicago 1951, P. 561, ASME ed. 1952.

A8. Houlihan, T. M., Thermowell Calculations, (Accuracy and Response), consulting report to Bechtel Power Corporation, October 1972.

A9. Nadai, A., Plasticity, McGraw-Hill, 1931.

A10. Nadai, A., Theory of Flow and Fracture of Solids, McGraw-Hill, 1950.

A11. Save, M. A., and Massonnet, C. E., Plastic Analysis and Design of Plates, Shells, and Disks, North-Holland Pub. Co., 1972.

A12. Timoshenko, S., Strength of Materials II, 3rd Ed., Section 70, p. 386 et seq., D. van Nostrand 1956.

A13. Ref. 12, pp. 96-99.

A14. Timoshenko, S., and Gere, J. M., Theory of Elastic Stability, McGraw-Hill, 1961, pp. 480-481.

APPENDIX B

ESTIMATION OF EXCITING FREQUENCY AND FORCES

TABLE OF CONTENTS

1. Estimation of Exciting Frequency and Forces

(Note: Except for very minor editorial emendations, this section is identical to Appendix B of Reference B2)

The study of the formation and shedding of vortices accompanying the flow of a fluid past a rigid circular cylinder held normal to the flow direction is an old one. A. W. Marris contributed an important review of this subject in 1964 (B4). A similar extensive treatment and discussion was given by J. H. Lienhard in 1966 (B3). The subject continues to be of great interest and importance. One of the reasons for this and for the fact that there has been funding for recent and current research is that large space vehicles, prior to launching, as they stand vertically on their launching pads, are subject to destructive action by horizontal terrestrial winds.

Analysis of a wide variety of experiments permits the following rather simple picture to emerge. A fixed, rigid cylinder, held normal to a fluid stream, distorts the flow of the latter. For most fluids under circumstances where boundary layers are formed and separation can take place there is periodic formation of vortices on the surface of the cylinder, arising from the separation of the boundary layer. The separation points move along the surface and eventually the vortex peels off (the vortex is shed by the cylinder) and proceeds downstream. This

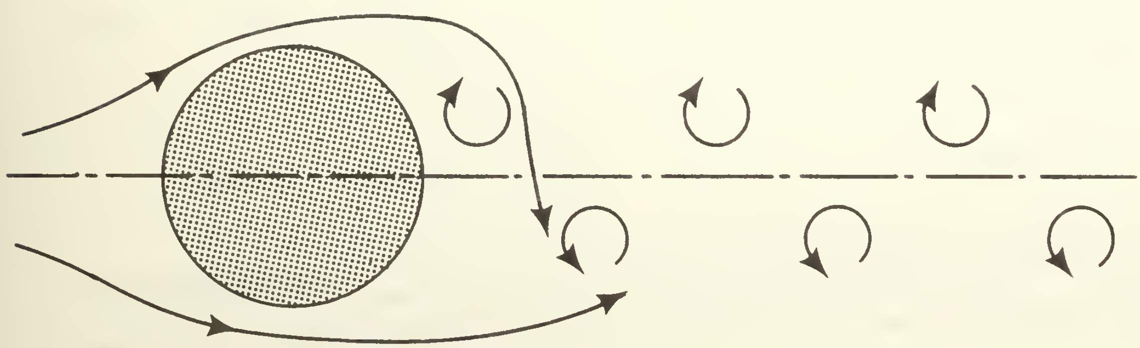


Figure B.1

phenomenon takes place alternately on the two sides of the cylinder as indicated in Figure B.1, which is adapted from Reference B3, and is accompanied by a drag force F_D, in the direction of the main stream flow, and a lift force F_L, normal to both main stream flow and cylinder axis. Forces F_D and F_L act upon the cylinder and cause its distortion if it is not perfectly rigid. If the displacement response of the cylinder is small, forces F_D and F_L appear as in Figure B.2.

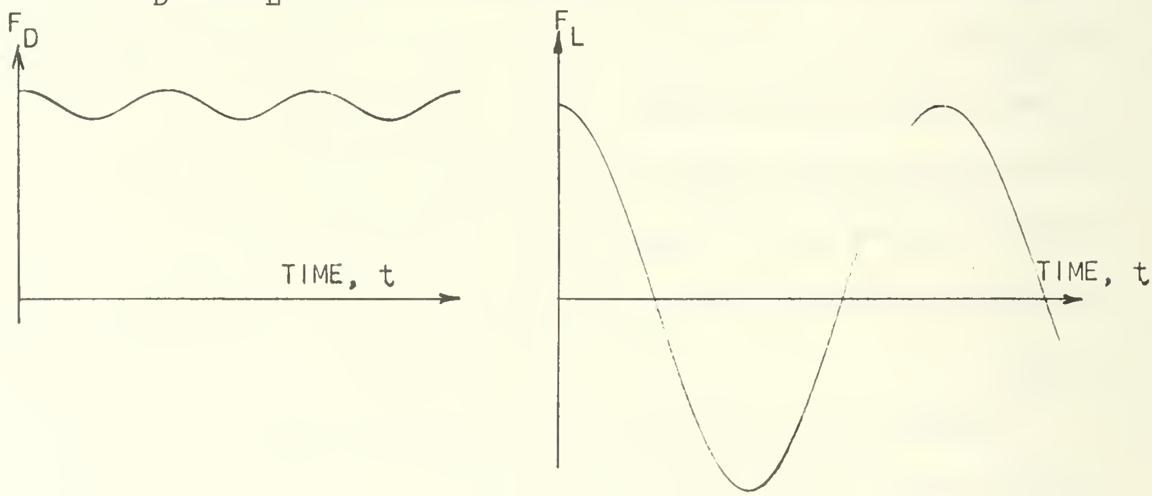


Figure B.2

The lift force, F_L, alternates in sense with a frequency f_s, called the Strouhal frequency, which we will discuss later; the mean value of F_L is zero. The drag force F_D is essentially constant with a small variable component at double the Strouhal frequency.

It is customary to represent these forces by use of so-called drag and lift coefficients, C_D and C_L, multiplied by a computed quantity having the dimensions of force. Thus we write

$$F_D = \frac{1}{2} C_D \rho A U^2; \qquad F_L = \frac{1}{2} C_L \rho A U^2 \qquad \text{(1a,b)}$$

where F_D is the mean magnitude of the drag force and F_L is the mean (half) amplitude of the lift force, U is fluid velocity upstream from the cylinder,

ρ is fluid mass density, and A is the projected area of the cylinder, namely length times diameter. Conveniently, we may consider only a unit length of cylinder in which case $A = D \times 1 = D$, and F_D and F_L are forces per unit length.

The Strouhal frequency f_s is representable by the formula

$$f_s = N_s\, U/D \tag{2}$$

where N_s, the Strouhal number, is a dimensionless quantity. The critically important quantities N_s, C_D and C_L vary depending upon flow conditions. Although the source and nature of the variations are not well understood, it provides a unifying viewpoint to consider their variations as depending upon the Reynolds number, N_R, given by

$$N_R = UD/\nu \tag{3}$$

where, as before U denotes undisturbed fluid velocity and D denotes cylinder diameter. The quantity ν denotes the kinematic viscosity of the fluid. (Do not confuse this with Poisson's ratio of an elastic solid, also represented by the symbol ν in Appendix A.) For water and steam, values may be obtained from the graphical presentation given in the ASME steam tables.

The variation of N_s, C_D and C_L with respect to N_R is large and complicated. Several distinct flow regimes exist for the range $10 < N_R < 10^7$. Lienhard illustrates and describes these; it should be made clear, however, that observation, understanding, and description is still far from clear or complete. See Lienhard (B3), p. 3.

At the time Murdock devised his analysis (1959) things seemed much simpler and more definite. For $N_R > 10^3$, for example, it was thought that $N_s = .21$ was almost a fact of nature. More recent work has shown how severe the variations are. The next illustration, Figure B.3, adapted from Lienhard (B3) and Popov (B6), shows the trends of these variations. The data available to Murdock was much less extensive, and, in particular, did not extend into the region of higher Reynolds numbers where the variation is most extreme and the data most scattered.

However, this region of large N_R is of particular interest for applications to steam power plant. For example, for Jim Bridger Main Steam system, $U = 230$ ft/sec., $\nu = .0054\ ft^2(sec \times 10^3)$ so that $N_R = 3.5 \times 10^6$. (Based on D = 1".). It is instructive to consider several of the important systems at this typical fossil fuel plant. See Table B-1 which shows that N_R for all the major systems lies in the range for which Murdock had no data and for which currently there is least information and greatest scatter.

It seems to be true that as N_R increases, N_s increases above the usually accepted value of 0.21 and, at about the same time the values of C_D and C_L decrease dramatically. Lienhard shows that the product of $C_D \times N_s$ is much more nearly constant than either alone. However, for our purposes, this knowledge is of little value since the information obtainable from these two parameters is utilized in quite different manners.

Thus, Murdock's assumptions (essentially $C_L = C_D = 1$, $N_s = .21$) should be reexamined in the light of more recent knowledge, Certainly Murdock was mistaken in believing that $C_L << C_D \approx 1.0$ so that taking $C_L = 1$ was grossly conservative, except possibly for $N_R < 10^4$. Our present information shows that they are roughly equal for $5 \times 10^4 < N_R < 10^6$. At the lower end of this range their roughly common value is about $C_D = C_L =$ 1.2 or 1.3 while at the upper end of the range the common value is about

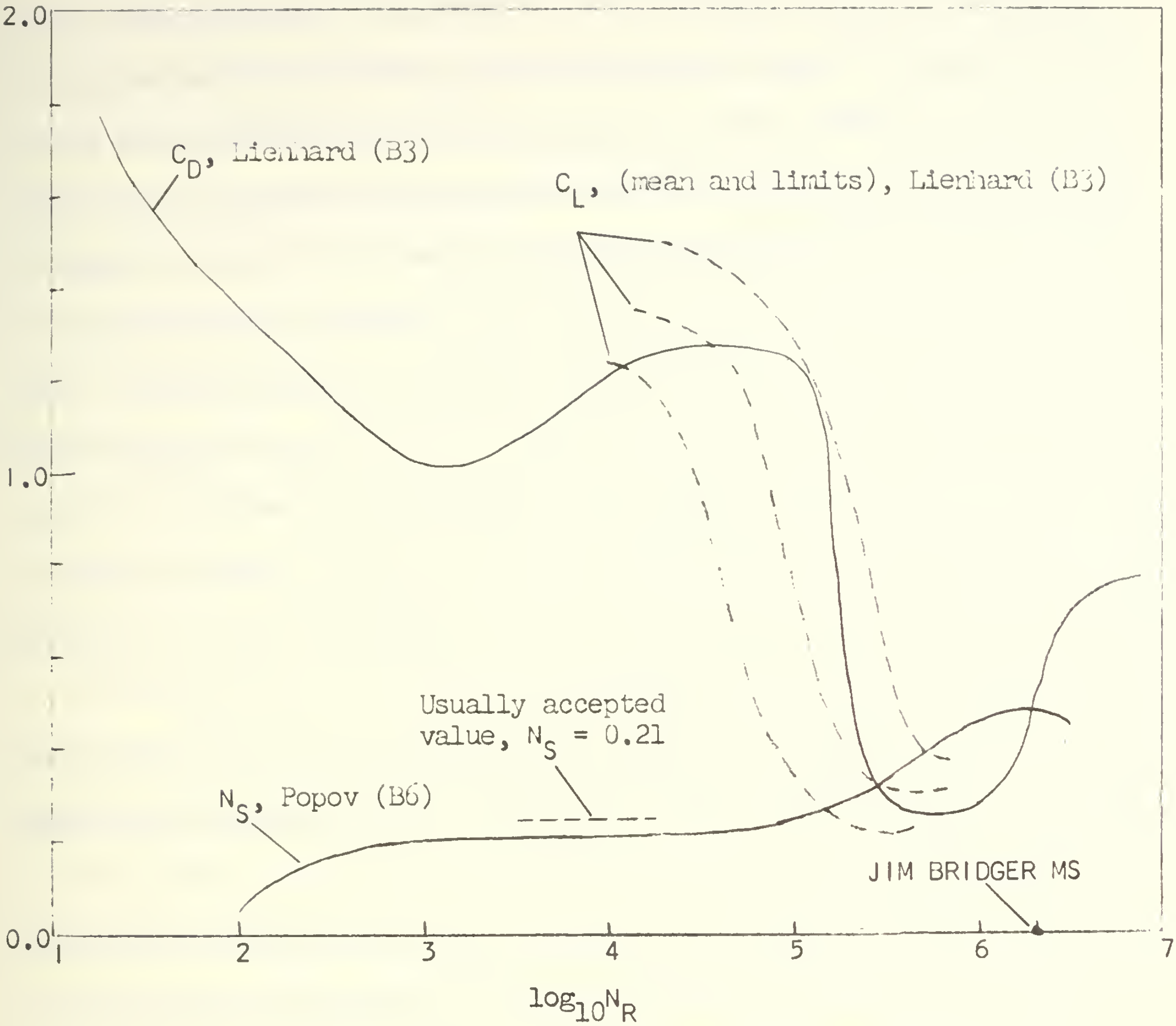


Figure B.3. Dependence of C_D, C_L, and N_s upon N_R adapted from Lienhard (B3) and Popov (B6)

TABLE B-1

N_R for Jim Bridger Systems*

System	T(^oF)	P, psig	U, ft /sec	$\nu, \frac{ft^2}{1000sec}$	D, ft	N_R
M.S.	1015	2520	230	.0054	1/12	3.55×10^6
H.R.H.	1015	627	182.5	.028	1/12	$.54 \times 10^6$
C.R.H	690	665	99	.015	1/12	$.55 \times 10^6$
B.F.D.	490	3720	24	.0015	1/12	1.33×10^6

*Data kindly supplied by Bechtel Power Corporation

0.2 or 0.3. This is indeed a drastic reduction. However, there is some difficulty in assuring that the value of N_R calculated by the formula $N_R = UD/\nu$ is indeed entirely appropriate since the flow through a pipe may differ in essential ways from the ideal flow conditions which were approximated in the experiments leading to Figure B.3. For one thing valves and bifurcations may be located close enough upstream that the assumption of uniform flow conditions upstream is quite incorrect. For another thing, the constriction provided by the pipe walls causes a slight speed-up of flow in the section which contains the thermowell. Thus one cannot be sure of the truly representative value of N_R, and one hardly has sufficient basis for knowing that, in a particular case, the values of C_D and C_L are indeed quite small.

Furthermore, Figure B.3 shows that N_s may be as high as .45 which is about twice the value N_s generally assumed for high speed flows and which is incorporated in Murdock's analysis.

Accordingly the following suggestion seems to provide a conservative procedure for purposes of strength analysis of thermowells. Calculate

$N_R = UD/\nu$

A. Strouhal number, upper (conservative) estimate

For $N_R < 4 \times 10^4$, take $N_s = .21$

For $4 \times 10^4 < N_R < 4 \times 10^5$, take $N_s = .24 \log_{10} N_R - .894$

For $N_R > 4 \times 10^5$, take $N_s = .45$

B. Drag coefficient, upper (conservative) estimate

For $3 \times 10^2 < N_R < 10^5$, take $C_D = 1.2$

For $N_R > 10^5$, take $C_D = .75$

C. Lift coefficient, upper (conservative) estimate

For $10^3 < N_R < 10^5$, take $C_L = 1.3$

For $N_R > 10^5$, take $C_L = .25$

I have discussed these values with my colleague, Dr. T. Sarpkaya, a recognized authority in this field and one who himself has made theoretical and experimental studies of flow past a cylinder, in particular with regard to the build-up to quasi-steady-state conditions. He has been good enough to look over the immediately preceding suggestions and to confirm that they adequately represent our present state of knowledge as applied to the engineering problem at hand, as being conservative in all cases but not extravagantly so.

For purposes of strength analysis, we will regard F_D as steady, neglecting its small variable component, and will regard F_L as sinusoidally varying at Strouhal frequency.

The discussion so far has presumed that the cylinder past which the flow is taking place neither deforms nor distorts. However, if there is significant deformation or distortion an unwelcome and destructive coupling may take place. This results from mechanical motion of the cylinder itself entering into and disturbing the flow field in such a way as to trigger the shedding of vortices. Thus, in the case of a cylinder which can vibrate as an elastic beam, as is indeed the case for a thermowell, which has, say, a well defined lowest natural frequency of elastic vibration, two modes of behavior may be distinguished. At low flow velocities, the Strouhol frequency is low ($f_s << f_n$). Excitation at f_s causes response at f_s, the magnitude of the response being generally small since the exciting forces are small. As U increases, f_s increases approaching f_n. The system is closer to resonance and the response (i.e., lateral displacement) increases. If f_s is sufficiently close to f_n, the response will be large enough to significantly influence the flow pattern and to interact with it. The frequency of vortex shedding approaches and "locks onto" the natural frequency f_n even though U does not increase. With excitation now taking place at precisely the natural frequency, a condition of mechanical resonance is attained in which amplitude of vibration builds up and is ultimately limited only by the damping which is present. If damping is insufficient, failure occurs quickly. If damping is sufficient to prevent early failure, still the material may suffer damage and fail by (high cycle) fatigue. Thus it is essential, regardless of whatever strength calculations may be made using C_D and C_L, to assure that f_s is sufficiently less than f_n to assure that this coupling is not significant and the locking or entrainment of frequencies phenomenon does not take place.

2. Discussion of and Addenda to the Preceding Section

Inasmuch as the criteria in the ASME Power Test Codes (A1), reflecting without change the criteria developed by J. W. Murdock, incorporate an unvarying value $\approx$0.21 of the Strouhal number, some surprise and consternation has been expressed at the introduction, in the recommendations in the writer's October 1972 report (B2), of values of N_s substantially larger than this. One very significant matter has been emphasized by more than one commentator. The experiments in the range of N_R for which these substantially larger values of N_s were obtained indicate a "randomness" or "lack of coherence" of the vortex shedding phenomena.

Expressed otherwise, the energy spectral density is sharply peaked at a frequency $f \approx .22U/D$ for $10^2 \leqq N_R \leqq 10^5$ and again fairly sharply peaked at $f \approx .29U/D$ for $10^7 \leqq N_R \leqq$ (?), whereas, for $10^5 \leqq N_R \leqq 10^7$ the energy spectral density is spread out in the range $.20U/D \leqq f \leqq .45U/D$. (The question mark above indicates uncertainty regarding an upper limit for the indicated restoration of coherence.) The question at hand is simply that of attempting to assess how much and what kind of damage may be done to a structure if the flow is characterized by N_R in the range $10^5 \leqq N_R \leqq 10^7$.

Because of lack of coherence it is reasonable to conclude that the chance of <u>sustained</u> excitation at any one particular frequency is slight, and the writer agrees with those who have pointed this out. This conclusion is reinforced by statements in a recent study (B1) which, in summarizing available literature of this date, points out also that there are phase differences in a spanwise direction under most circumstances, and these themselves become incoherent when the shedding becomes incoherent. Another, as yet unpublished study of a classfied project (the writer must certainly apologize for adducing such a nebulous source) indicates that significant structural excitation of certain test structures did not take place (for flows in the regime presently under discussion) in a rather extensive series of experiments. Accordingly, it does indeed seem perfectly clear that coherent excitation, of the kind possible for $10^2 < N_R < 10^5$ and for $N_R > 10^7$, does not occur for $10^5 < N_R < 10^7$.

Accordingly, it is appropriate to consider the worst that <u>could</u> occur (if $10^5 < N_R < 10^7$) and the probability of its occurrance. Anyone who has ever seen them cannot ever forget the moving pictures, now about thirty years old, of the tail structures of certain WWII aircraft when aerodynamic flutter occurred: one oscillation, two oscillations, three oscillations, -- GONE! In the present application we may ask how many coherent, in-phase excitations can lead to dangerous displacement excursions. More than three, certainly, but how many? One hundred?

One hundred cycles at a natural frequency of approximately 3000 Hz (a typical value), occurs in 30 milliseconds. What are the chances of 30 milliseconds coherence in a twenty year design life?

Thus, we have two questions which certainly we cannot answer. How many coherent cycles will result in damage?, and what is the probability of getting these cycles sometime during a twenty year or thirty year design lifetime.

Now a logical engineering attitude is to presume that damage can occur from this source, however unlikely that may seem to be, if the cost of doing so is not too great. In other words, we propose to include criteria such as to assure that this kind of potentially damaging situation does not arise. If this costs nothing as a practical matter it does no harm that we may indeed be "over safe". If there is an implied penalty, then one is perfectly free to violate the criterion, but only with the understanding that the possibility of structural damage has been increased. A designer might take different courses depending on whether the thermowell involved was in a system in a fossil fuel plant or in a nuclear plant where failure could carry undesirable material into a radioactively "hot" zone.

Criteria intended to assure no possibility whatsoever of damage of this kind were included in the recommendations of (B2). However, realism requires adding other criteria upon which one may fall back if the original criteria are felt to be unduly restrictive or uneconomical in a particular situation. The secondary criteria should be related to a greater degree of risk, but one which is economically acceptable under most circumstances.

However, the state of our knowledge is simply not adequate for a quantitative assessment of risk. Accordingly, all that can reasonably be suggested at this time is to add to the recommendations made two years ago, an explanatory note, at the proper place or places and to the following effect:

If $N_R > 4 \cdot 10^4$ and the design meets Criterion No. 1 but fails to meet Criteria 2, 3, or 4, repeat the calculations for the latter, but using the artificial value $N_R = 4 \cdot 10^4$. If all criteria are now satisfied, the design is acceptable except for those cases where an unusually great penalty would be associated with failure. If one or more criteria remain unsatisfied, the design may still be acceptable, but special calculations are required to show this. Such calculations should be based upon the general analytical procedures in this report but may employ, in a consistent manner, whatever appropriate experimental information may be available at the time of the calculations.

The last provision in the preceding paragraph takes cognizance of the fact that the probability is that the regime $10^5 \leqq N_R \leqq 10^7$ is indeed safer than other regimes since not only is the excitation incoherent but also the value of C_D and C_L have decreased. One should not become confused by this apparent disparate use of the word "safer". In other regimes (than $10^5 < N_R < 10^7$) there is a reasonable degree of certainty in calculating the exciting frequency and forces. In the regime $10^5 < N_R < 10^7$ the degree of certainty is significantly less. The "odds are" that the danger of damage is less, but the certainty that this is so is significantly less. An analog may help explain this. For N_R outside the range $10^5 - 10^7$, we could say that we <u>expect</u> to lose 100 tokens but could lose as much as 200. For N_R within the range $10^5 - 10^7$, we expect to lose 10 tokens but could lose as much as 1000.

3. Bibliography for Appendix B

B1. Barnett, K. M., and Cermak, J. E., Turbulence induced changes in vortex shedding from a circular cylinder, Project THEMIS Technical Report No. 26, College of Engineering, Colorado State University, Fort Collins, Colorado, January 1974.

B2. Brock, J. E., Stress analysis of thermowells, consulting report for Bechtel Power Corporation, October 1972, and supplement, December 15, 1972.

B3. Lienhard, J. H., Synopsis of lift, drag, and vortex frequency data for rigid circular cylinders, Bulletin 300, Washington State University College of Engineering, Research Division, Pullman, Washington, 1966.

B4. Marris, A. W., A review on vortex sheets, periodic wakes, and induced vibration phenomena, J. Basic Engrg, June 1964, pp. 185-196.

B5. Morkovin, M. V., Flow around circular cylinder -- a kaleidoscope of challenging fluid phenomena, Proceedings of the ASME Symposium on Fully Separated Flows, Philadelphia, pp. 102-118. ASME 1964.

B6. Popov, S. G., Relation between Strouhol and Reynolds numbers for two-dimensional flow about a circular cylinder, Fluid Dynamics, pp. 107-109 (trans. from Mekhanika Zhidkosti i Gaza, Vol. 1, No. 2, pp. 156-159, 1966.

Note: Reference B5, which is not cited in the text hereof, says much the same as several of the other references. It was duplicated and sent to the ASME PB 51 Committee through the kindness of Mr. J. W. Murdock and Mr. W. O. Hayes.

APPENDIX C

RESPONSE FREQUENCIES OF THERMOWELL VIBRATION

TABLE OF CONTENTS

1. Introduction

Previous analyses of thermowell vibration have, reasonably, focussed on "cantilever" vibrations in which the thermowell is considered fixed at its root and is subject to flexural vibrations in a single transverse plane. If the thermowells were longer and more slender than they actually are, there would be no serious difficulty in making reasonable estimates of response frequency. The non-uniformity of cross section would introduce only minor difficulties.

However, there are basic difficulties of a more serious nature. First, for short, stubby cantilever beams the so-called elementary beam theory does not take into account what may be a significant elastic compliance, namely that due to shear deformation. Second, for such stubby beams, the usual dynamic analysis does not take into account what may be a significant inertial effect, namely that due to longitudinal motion of the mass particles of which the beam is composed.

A generally accepted procedure for accounting for these two effects, both of which tend to depress the response frequencies as compared to values computed on the basis of elementary theory, is by use of so-called Timoshenko beam theory which takes into account shear deflection and rotatory inertia. This theory is still approximate but is widely believed to provide results of sufficient accuracy for engineering purposes, particularly for the lowest response frequencies. This theory is well established and many applicable and useful results are available. These will be discussed later in this Appendix.

A much more troublesome difficulty is that associated with the assumption of root end restraint. The body (in our case, the pipe wall) to which the cantilever is fixed at its root end is not perfectly rigid.

Accordingly, the assumptions regarding root end fixity which are invariably a part of cantilever response analysis are simply not true. For long slender cantilevers, the resulting errors are acceptably small, but for short, stubby cantilevers, the degree of error may be appreciable. The foundation yields, so that the "system" composed of beam and foundation has increased compliance. Furthermore, the foundation possesses nonzero mass. Both of these considerations tend to decrease response frequency.

Thus there are four nonelementary influences each of which results in a decrease of frequency as compared to results estimated using elementary theory. They are: (a) shear deflection in the beam, (b) elastic compliance of the foundation, (c) rotatory inertia in the beam, and (d) mass-inertia in the foundation.

The major attention in this Appendix will be focussed on methods of accounting for these influences. However, one additional aspect must be considered. Along with modes which can be roughly described as cantilever modes, there is a low-frequency mode in which the pipe itself is periodically ovalized and in which the thermowell acts more or less as a rigid body. This possibility has not been treated in thermowell analyses except in a previous development by this writer. Section 14 of this Appendix C will deal with this mode of vibration.

2. Finite Element Analysis

The analytical and other difficulties surrounding the business of estimating the response frequencies of thermowells - in particular the lowest or slowest frequency which is of the greatest interest - are so great that no strictly analytical procedure is presently developed to the point where it can account for all the effects. However, there is a method, designated as FEM, an abbreviation for "finite element method," which has proved its power and has attracted much attention and employment in the past few years, and which seems to be a "natural" for the problem at hand. A former student of the writer, Mr. J. R. Adamek, undertook to employ FEM to study thermowell vibrations, but found it advisable to divert his attention to the development of mesh generation software for the purpose of making available FEM software easier to use. Currently Mr. H. L. Crego is engaged in dealing with the thermowell vibration problem using FEM. His program contemplates first dealing with a two-dimensional formulation which definitely is not representative of the real thermowell problem but which is more easily attacked since computer system limitations are less severe with two-dimensional than with three-dimensional problems. Following successful treatment of the 2D problem an attack will be made on the 3D problem.

Mr. Crego is modifying for his particular use the software programs PLISOP and PLIMEG previously developed for 2D problems. For the reader who has some familiarity with FEM analysis the following steps - some already complete in August 1974 - may be of interest. PLIMEG and PLISOP are being united. In addition to the generation by PLISOP of the consistent stiffness matrix K, the new facility of generating a consistent mass matrix M has been developed. Matrices M and K are banded and symmetrical which permits compact storage. A triangular decomposition of K will be made, preserving compactness of storage. The iterative algorithm

$$Kv_{n+1} = \omega^2 Mv_n \qquad (1)$$

will be employed. The triangular decomposition of K is, in effect, a "forward solution" and the solution for each improved vector v_{n+1} will involve only a "back solution" which is very rapidly accomplished. A standard normalization will be employed to find the new vector and to estimate ω^2. It is also contemplated to use a final Rayleigh evaluation

$$\omega^2 = v^T K v / v^T M v \tag{2}$$

so as to reduce the number of iterating steps. Since only $\omega_1{}^2$ is of interest there is no need to purge or filter to permit obtaining higher modes.

No difficulties are contemplated in implementing the program. However, considerable experimentation will have to be done to evaluate the influence of "how much foundation" is included in the formulation and of the constraints on this portion. Furthermore, it is anticipated that the 2D and 3D problems may differ greatly in this respect. It is not contemplated, presently at least, to represent the foundation as other than a rectangular mass of material. In particular, it is not presently contemplated to model the pipe wall as a tube.

3. Foundation Compliance

Because of the importance of foundation compliance and inertia, ideally the problem at hand is one of simultaneously dealing with the beam (thermowell) and the foundation (pipe wall) in what is known (in heat transfer theory terminology) as a conjugated problem. Leaving aside the matter of damping (energy sinks) one assumes isocronous vibration with common frequency ω of both the beam and the foundation. At their interface the displacements must match and the stress components must conform. This problem is utterly beyond the reach of analytic procedures and is accessible only to "subdividing" techniques such as FEM. Accordingly, only approximate approaches are possible.

The foundation behavior presents the greater difficulties for analytic treatment. Inasmuch as we presently have no real idea at all of the quantitative effect of foundation behavior it is reasonable to look for whatever results we can get for foundation performance. Thus, at the outset we must abandon the question of the effect of foundation inertia and settle for what we can evolve concerning foundation compliance.

The literature contains a number of approximate evaluations of foundation compliance: (7), (11), (17), (20), and (21). All of these, except only (7), deal with a 2D situation. Although in an earlier analysis (6), the writer claimed that the 3D situation of the thermowell-pipe problem lay somewhere between the plain stress and the plane strain 2D cases in the literature, now he concludes that such is not the case at all, and that only a 3D analysis is applicable.

There are two classical problems of the analysis of a semi-infinite solid subject to the action of a force applied to a point on its surface. In Boussinesq's problem, the force $P\bar{k}$ at the origin, is applied to the solid $z \geqq 0$ and the vector displacement $\bar{\rho}$ at the point $\bar{r} = \bar{i}x + \bar{j}y + \bar{k}z$ is given by

$$\begin{aligned}\bar{\rho} = {} & (P/4\pi Gr)\{[z/r^2-(1-2\nu)/(r+z)]\bar{r} \\ & + [3z-4\nu z+2(1-\nu)r]\bar{k}/(r+z)\}\end{aligned} \tag{3}$$

where G denotes the shearing modulus, ν denotes Poisson's ratio and $r = |\bar{r}|$. On the surface, where $z = 0$, this becomes

$$\bar{\rho}(\text{surface}) = (P/4\pi Gr^2)[-(1-2\nu)\bar{r} + 2(1-\nu)r\bar{k}] \tag{4}$$

A similar problem, bearing the name of Cerruti, is the same except that the force applied to the origin is $P\bar{i}$ rather than $P\bar{k}$, and the displacements are given by

$$\bar{\rho} = (P/4\pi Gr)\{\bar{i}+x\bar{r}/r^2+(1-2\nu)[r(z\bar{i}+x\bar{k}) + (x^2+y^2)\bar{i}-xy\bar{j}+xz\bar{k}]/(r+z)^2\} \tag{5}$$

$$\bar{\rho}(\text{surface}) = (P/4\pi Gr^3)\{2[x^2+(1-\nu)y^2]\bar{i} + 2\nu xy\bar{j} + (1-2\nu)rx\bar{k}\} \tag{6}$$

These solutions are presented conveniently by Westergaard (25). Love (16) discusses these and related problems. It is noteworthy that Timoshenko, one of the foremost elasticians to write in the English language, seems nowhere to mention Cerruti's work; cf. esp. (24).

Now a rational approach to estimating foundation stiffness or compliance is to apply a "reasonable" distribution of normal and shearing forces to the interface area of the semi-infinite solid, use the formulas above to calculate surface displacements, and, by defining suitable averages, infer constraint rotation and displacement. A reasonable distribution of normal forces is the linear distribution given by the elementary formula $\sigma = Mx/I$ and a reasonable distribution of shearing forces is that given by the elementary formula $\tau = VQ/Ib$.

In this way, one could estimate the coefficients $b_{M\theta}$, $b_{M\Delta}$, $b_{V\theta}$, and $b_{V\Delta}$ giving compliance coefficients for rotation (subscript θ) and deflection (subscript Δ) corresponding to unit moment (subscript M) and unit shear (subscript V).

Brown and Hall (7) have made an evaluation of $b_{M\theta}$ obtaining the value

$$b_{M\theta} = (16/15\pi)(1-\nu^2)d/EI \approx .787/Ea^3 \tag{7}$$

for a circular interface of radius a. This evaluation is based solely upon the $\bar{k}$ component of $\bar{\rho}$ (surface) of Equation (4), i.e.

$$\bar{k} \cdot \bar{\rho} \text{ (surface)} = 2(1-\nu)P/4\pi Gr = P(1-\nu^2/\pi Er \tag{8}$$

The writer has recently made an estimate of $b_{V\Delta}$ from Cerruti's analysis, proceeding as follows. Using only the $\bar{i}$ component from Equation (6), viz

$$\bar{i} \cdot \bar{\rho}\text{(surface)} = (P/2\pi Gr^3)[x^2+(1-\nu)y^2] \tag{9}$$

and computing dF from the VQ/Ib formula,

$$dF = 4P(a^2-r^2\cos^2\theta)\,r\,dr\,d\theta/3\pi a^4 \tag{10}$$

we calculate the displacement of the center of the circular interface, of radius a, to be

$$\begin{aligned}\Delta &= (4P/6\pi^2Ga^4)\int_0^{2\pi}\int_0^a (a^2-r^2\cos^2\theta)(1-\nu-\nu\cos^2\theta)\ dr\ d\theta \\ &= P(20-11\nu)/18\pi Ga = P(20-11\nu)(1+\nu)/9\pi Ea \simeq 0.768P/Ea\end{aligned} \tag{11}$$

so that

$$b_{V\Delta} = 0.768/Ea \tag{12}$$

We have made no effort to obtain an average deflection. This is the deflection at the center of the circular area, which seems to be the appropriate quantity for our purposes.

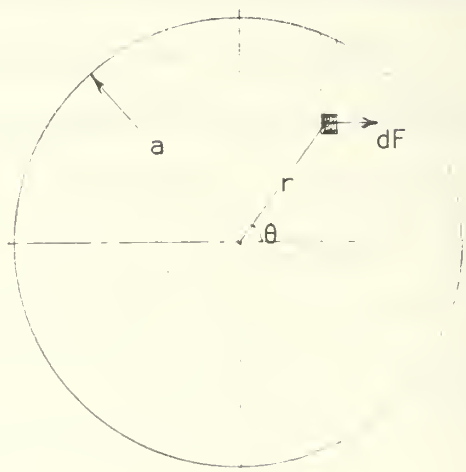


Fig. C.1 Figure to assist in establishing Equation (11).

However, our attempts to evaluate $b_{M\Delta}$ and $b_{V\theta}$ have not been successful because of analytical difficulties in evaluating the appropriate integrals, which are not difficult to set up. We recommend this problem to students who are searching for a useful problem area to which to contribute.

The results given in Equations (7) and (12) appear to be the only results truly applicable to the problem at hand. The integrals defining $b_{M\Delta}$ and $b_{V\theta}$ almost certainly do not vanish. However, their values are simply not presently available. Thus, for present purposes we can do no better than to take

$$b_{M\theta} = .787/Ea^3;\ b_{V\theta} = b_{M\Delta} = 0;\ b_{V\Delta} = .768/Ea \qquad (13)$$

4. Beam Vibration Equations

We will here derive quite general equations for beam vibration including the effect of influences of little or no concern with regard to thermowells. The reason for doing this is to present in this public document a general description which may be used for other purposes. We will thus include shear deflection, rotatory inertia, elastic (Winkler) support, an axial compressive load P, and two kinds of damping.

Figure C2 shows a beam element in its deflected position and configuration. We use $\dot{} = \frac{\partial}{\partial t}$, $' = \frac{\partial}{\partial x}$. V and M represent actual shear force and bending moment exerted by the material to the left of the element; $(V + V'dx)$ and $(M + M'dx)$ represent these efforts on the right. y denotes the vertically upward deflection of the center of the element, and y' represents the slope of the locus of such centers. ψ represents the slope, away from the vertical, of the face of the element, in the deflected position. $\gamma\ddot{\psi}dx$ is a

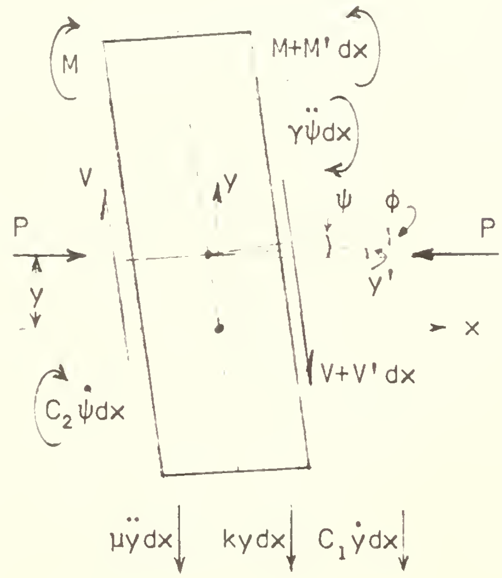


Fig. C.2 General beam element with real and D'Alembert forces and moments.

D'Alembert moment due to rotatory inertia; γ is the mass moment of inertia per unit length. $C_2\dot{\psi}dx$ is a moment resulting from an internal damping property and $C_1\dot{y}dx$ is a force resulting from external damping. $\mu\ddot{y}dx$ is a D'Alembert force due to lateral deflection, μ being mass per unit length, and kydx represents a Winkler restoring force. We presume that angles y', ϕ, and $\psi = y' + \phi$ are small. Under these conditions, and assuming elastic behavior, the following equations are obtained

$$M' = V + \gamma\ddot{\psi} + C_2\dot{\psi} \qquad \text{(Dynamic equilibrium)} \qquad (14)$$

$$V' = -\mu\ddot{y} - ky - C_1\dot{y} \qquad \text{(Dynamic equilibrium)} \qquad (15)$$

$$\psi = y' + \phi \qquad \text{(Geometry)} \qquad (16)$$

$$EI\psi' = M - Py \qquad \text{(Elasticity)} \qquad (17)$$

$$\phi = V/k_sAG \qquad \text{(Elasticity)} \qquad (18)$$

E is Young's modulus, G is shear modulus, A is cross sectional area, I is second moment of A about the controidal axis, and k_s is a shearing force distribution factor. The rotatory inertia term $\gamma\ddot{\psi}$ neglects distortion of cross section; this is a part of so-called Timoshenko beam theory. It is convenient to write

$$B = EI, \; b = (k_sAG)^{-1} \qquad (19a,b)$$

In general, these are functions of position but not of time. The same is true for k, γ, C_1, and C_2.

5. Constant Section and Properties

Usually, textbook derivations confine attention to cases where there is no variation of shape or properties. In this case, it is very easy to obtain the equations

$$By'''' + BbV''' + Py'' = V' + \gamma\ddot{\psi}' + C_2\dot{\psi}' \qquad (20a)$$

$$V' = -\mu\ddot{y} - ky - C_1\dot{y} \qquad (20b)$$

$$V''' = -\mu\ddot{y}'' - ky'' - C_1\dot{y}'' \qquad (20c)$$

Taking the state vector to be

$$Y = [y, y', M, V]^T \tag{27}$$

there is no difficulty in determining the (transfer) matrix

$$U = VDV^{-1} \tag{28}$$

where

$$V = \begin{bmatrix} 1 & 1 & 1 & 1 \\ m_1 & m_2 & m_3 & m_4 \\ Bm_1{}^2 & Bm_2{}^2 & Bm_3{}^2 & Bm_4{}^2 \\ Bm_1{}^3 & Bm_2{}^3 & Bm_3{}^3 & Bm_4{}^3 \end{bmatrix} \tag{29}$$

$$D = \begin{bmatrix} \exp(m_1 x) & 0 & 0 & 0 \\ 0 & \exp(m_2 x) & 0 & 0 \\ 0 & 0 & \exp(m_3 x) & 0 \\ 0 & 0 & 0 & \exp(m_4 x) \end{bmatrix} \tag{30}$$

$$B = \quad EI \tag{19a}$$

The transfer matrix U is such that

$$Y_{RIGHT} = UY_{LEFT} \tag{31}$$

and this is its principal property and employment.

Although it is of some interest to obtain and exhibit a formula for V^{-1}, which can be done easily by considering an interpolation problem, as a practical matter in the employment of transfer matrix theory, corresponding to any choice of value of ω, it is simplest to obtain the inverse numerically, employing any complex arithmetic inversion procedure.

The case where the roots m_j are not distinct need not actually be faced when one is using a numerical procedure. Likewise, for present

Substituting (20b) and (20c) into (20a), we get

$$By'''' - (\gamma+Bb\mu)\ddot{y}'' - (C_1Bb+C_2)\dot{y}'' + (P-kBb)y'' + \gamma b\mu\ddddot{y}$$
$$+ b(\gamma C_1+\mu C_2)\dddot{y} + [b(\gamma k+C_1C_2)+ \mu]\ddot{y} + (C_1+C_2bk)\dot{y} + ky = 0 \qquad (21)$$

If we look for an isochronous solution

$$y(x,t) = y(x)\ \exp(i\omega t) \qquad (22)$$

we get

$$By'''' + [P-kBb+\omega^2(\gamma+Bb\mu)-i\omega\ (C_1Bb+C_2)]y''$$

$$+ \{[k-\omega^2(\mu+b\gamma k+bC_1C_2)+ \omega^4\gamma b\mu] + i\ [\omega(C_1+C_2bk)- \omega^3b(\gamma C_1+\mu C_2)]\}y = 0$$

Dividing by B we get

$$y'''' + \alpha y'' + \beta y = 0 \qquad (23)$$

where α and β depend on frequency, and, unless $C_1 = C_2 = 0$, are complex

The transfer matrix corresponding to a beam element of this kind may be obtained as follows. Substituting

$$y = a\ \exp\ (mx) \qquad (24)$$

we get the indicial equation

$$m^4 + \alpha m^2 + \beta = 0 \qquad (25)$$

(In general) there will be four roots $m_j(j = 1,\cdots, 4)$ which are complex numbers. The solution, then, is

$$y(x) = \Sigma a_j\exp(m_jx) \qquad (26a)$$

$$y'(x) = \Sigma a_jm_j\exp(m\ x) \qquad (26b)$$

$$M = EIy''(x) = EI\Sigma a_jm_j{}^2\exp(m_jx) \qquad (26c)$$

$$V = EIy'''(x) = EI\Sigma a_jm_j{}^3\exp(m_jx) \qquad (26d)$$

purposes, there is no need to consider obtaining the solution to the non-homogeneous equation. However, both these courses can be pursued without any essential difficulty since the theory of linear differential equations with constant coefficients is so thoroughly developed.

6. Non-constant Section and/or Properties

Unless the section and properties are constant, it is not possible to reduce Equations (14) - (18) incl. to as simple and useful a form as (21) or (23). A numerical procedure could probably be devised to deal with Equations (14) - (18) directly. However, any such procedure would be a "lumping" procedure equivalent to or nearly equivalent to the assumption of piecewise constancy. Accordingly, one can as well assume piecewise constancy and employ transfer matrix methodology throughout. For our cantilever beams, we would take

$$Y_{Root} = [0,0,y_R'',y_R'''] \tag{32a}$$

$$Y_{Tip} = [y_T,y_T',0,0] \tag{32b}$$

and represent the transfer matrix between root and tip by a continued product of transfer matrices of the type shown in Equation (28), each for an assumed uniform sub-length of the beam.

There seems to be no intrinsic difficulty in producing a computer program capable of dealing with this problem. As a long term project the writer hopes to do this, including in the program a catalog of transfer matrices for interesting elements other than beam sections. One such program, with no dynamic capability, has already been successfully used to determine Euler buckling loads for nonuniform columns. The application and motivation for this program was originally to deal with the "squirming" of straight pipe assemblies containing bellows expansion "joints" and details of the computation are proprietary to Tube Turns Division of Chemetron Corporation, for which the writer performed this work several years ago.

7. Alternate Approach

However, for the present purpose, namely that of making a "reasonably accurate" determination of (lowest) natural frequency of cantilever-like vibrations of thermowells, it does not seem necessary to apply such a sophisticated procedure. An alternate viewpoint may be described as follows.

Use "ordinary" beam theory. Omit all damping, rotatory inertia, and shear deflection. Also note that in this problem there is no Winkler support and the effect of axial force P may be neglected. The resulting simplified and idealized situation is easily dealt with. Murdock's analysis used a Rayleigh type of approximation (employed in a somewhat less general way than the present writer would have preferred). The writer prefers a Stodola type of solution for the problem at hand. However, properly used these, and other, methodologies lead to the same numerical determinations with a quite tolerable margin of approximation.

Then, having the "simple beam theory" evaluation for (lowest) natural frequency, one attempts to estimate the effect of the foundation and of shear deflection and rotatory inertia. In doing the latter, attention is focussed upon their exactly calculable effect in the case of uniform beams and it is reasonably assumed that a similar (qualitatively and quantitatively) effect is applicable in the case of nonuniform beams. The errors in doing this are surely less than other errors of idealization, principally those concerned with the degree of root compliance.

A number of studies have been made comparing frequencies calculated accounting for shear deflection and rotatory inertia (8), (9), (10), (14), (15) and some of these explicitely deal with linearly tapered cantilevers of circular cross section. These latter results would be directly applicable to our problem except for two things. First, thermowells have a central cavity which subtracts from their mass (as compared to conical frustra) and, to a smaller extent, from their stiffness. Second, all these studies presume perfectly rigid root constraint.

While one can argue that neglecting the central cavity causes under-evaluation of natural frequencies which is conservative (safe) for our purposes, neglecting the realities of root support causes over-evaluation of natural frequencies which is nonconservative. Accordingly, these results cannot be accepted as directly applicable to the problem at hand.

8. "Assembly" of Analytic Procedures

The foundation compliances b_{ij} of Section 3 of this Appendix may be incorporated in a "foundation transfer matrix"

$$U_F = \begin{bmatrix} 1 & 0 & b_{M\Delta} & -b_{V\Delta} \\ 0 & 1 & b_{M\theta} & -b_{V\theta} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{33}$$

so that the entire structure, including foundation, can be represented by the transfer matrix

$$U = U_N \, U_{N-1} \cdots U_3 \, U_2 \, U_1 \, U_F \tag{34}$$

where U_k is representative of a kth section, assumed uniform, proceeding from the left. In general, U_k will, as we will see, depend upon sinusoidal frequency ω. Also, U_F should depend on ω because of the contribution of its inertia; however, our evaluation has not been sufficiently thorough as to take foundation mass into account; furthermore, for lack of knowledge, we must take $b_{M\Delta} = b_{V\theta} = 0$.

Then the transfer matrix procedure of determining natural frequencies consists of dealing with

$$Y_{RIGHT} = UY_{LEFT} \tag{35}$$

where it is known that

$$Y_{RIGHT} = [y_R, y_R', 0, 0] \tag{36}$$

i.e., a "free" condition, and

$$Y_{LEFT} = [0, 0, M_L, V_L] \tag{37}$$

i.e., a "clamped" condition.

If we write

$$U = \begin{bmatrix} u_{11} & u_{12} & u_{13} & u_{14} \\ u_{21} & u_{22} & u_{23} & u_{24} \\ u_{31} & u_{32} & u_{33} & u_{34} \\ u_{41} & u_{42} & u_{43} & u_{44} \end{bmatrix} \tag{38}$$

the condition becomes

$$\begin{bmatrix} u_{33} & u_{34} \\ u_{43} & u_{44} \end{bmatrix} \begin{bmatrix} M_L \\ V_L \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{39}$$

and for a solution other than $M_L = V_L = 0$, we must have

$$\begin{vmatrix} u_{33} & u_{34} \\ u_{43} & u_{44} \end{vmatrix} = u_{33}u_{44} - u_{34}u_{43} = 0 \tag{40}$$

and the natural frequencies ω_1, ω_2 ... must be such as to satisfy this relation. (This is the essence of transfer matrix analysis for natural frequencies.)

Transfer matrices U for other than uniform beam elements are, at best, quite complicated. Accordingly, as indicated above, our procedure could be to consider a nonuniform beam as a concatenation of piecewise uniform segments. In this way we can construct a procedure (a digital computer program) capable of dealing (to a satisfactory degree of approximation if section changes are "gradual") with a general, nonuniform cantilever having root compliance and taking shear deflection and rotatory inertia into account.

9. Calculations for Uniform Cantilever

As has been indicated above, a program for determining cantilever-type frequencies can be constructed, accounting for elementary effects and the additional complications due to non-uniformity, shear deflection, rotatory inertia, and foundation compliance, although, as has been pointed out, our knowledge of appropriate foundation parameters is incomplete and faulty in that it does not consider foundation inertia.

However, this program has not yet been written. Moreover, for practical daily design use a simpler viewpoint is to be preferred. Accordingly we now investigate the case of a uniform cantilever, taking into account, one after another, the non-elementary complications listed above. We will see how great their influence is and will suggest that the frequencies of non-uniform cantilevers will be modified in the same way and roughly to the same extent.

Thus we want an appropriate transfer matrix U for a uniform cantilever, and we adapt this from the information given on page 136 of (22). In accordance with the sign conventions indicated in Fig. 3-10, page 55 of (22), all four elements of our present state vector are the negatives of the corresponding elements of the state vector shown there. Accordingly no changes of sign are required in the transfer matrix, which is

$$U = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \tag{41}$$

where

$$\begin{aligned} &a_{11} = c_o - \sigma c_2,\ a_{12} = \ell[c_1 - (\sigma+\tau)c_2],\ a_{13} = \ell^2 c_2/EJ \\ &a_{14} = \ell^3[-\sigma c_1 + (\beta^4+\sigma^2)c_3]/\beta^4 EJ,\ a_{21} = \beta^2 c_3/\ell,\ a_{22} = c_o - \tau c_2, \\ &a_{23} = \ell(c_1 - \tau c_3)/EJ,\ a_{24} = \ell^2 c_2/EJ,\ a_{31} = \beta^4 EJ c_2/\ell^2, \\ &a_{32} = [-\tau c_1 + (\beta^4+\tau^2)c_3]EJ/\ell,\ a_{33} = c_o - \tau c_2,\ a_{34} = \ell[c_1 - (\sigma+\tau)c_3] \\ &a_{41} = \beta^4 EJ(c_1 - \sigma c_3)/\ell^3,\ a_{42} = \beta^4 EJ c_2/\ell^2,\ a_{43} = \beta^4 c_3/\ell,\ a_{44} = c_o - \sigma c_2 \end{aligned} \tag{42}$$

and

$$\begin{aligned} c_o &= \Lambda(\lambda_2{}^2 \cosh\lambda_1 + \lambda_1{}^2 \cos\lambda_2) \\ c_1 &= \Lambda(\lambda_2{}^3 \sinh\lambda_1 + \lambda_1{}^3 \sin\lambda_2)/\lambda_1\lambda_2 \\ c_2 &= \Lambda(\cosh\lambda_1 - \cos\lambda_2) \\ c_3 &= \Lambda(\lambda_2 \sinh\lambda_1 - \lambda_1 \sin\lambda_2)/\lambda_1\lambda_2 \\ \sigma &= \mu\omega^2\ell^2\kappa/AG, \tau = \mu i^2\omega^2\ell^2/EJ, \quad \beta^4 = \mu\omega^2\ell^4/EJ \\ \lambda_{1,2} &= \sqrt{\sqrt{\beta^4 + (\sigma-\tau)^2/4} \pm (\sigma+\tau)/2},\ \Lambda = (\lambda_1{}^2 + \lambda_2{}^2)^{-1} \end{aligned} \tag{43a....j}$$

Here E and G are physical properties of the material, κ is a shear form factor, μ is mass per unit length, ℓ is length, J is the moment of inertia of the cross section about a centroidal axis perpendicular to the plane of vibration, and i is the radius of gyration about the same axis. Note that $J = Ai^2$.

It is important to note that μ includes not only the mass of the cantilever itself but also the "added" mass of the fluid which may be considered to move with it. For the case of a cylinder under stationary conditions, the added mass may be shown to be equal to the mass of the displaced fluid and this assumption should be made here.

Writing

$$U_{33} = U_{41}\ U_{33} \tag{44}$$

where the subscripts refer to equation numbers, and defining (compare equation (40))

$$\Delta = u_{33}\ u_{44} - u_{34}\ u_{43} \tag{45}$$

we get

$$\begin{aligned}\Delta = &(a_{31}b_{M\Delta}+a_{32}b_{M\theta}+a_{33})(-a_{41}b_{V\Delta}-a_{42}b_{V\theta}+a_{44}) \\ &-(a_{41}b_{M\Delta}+a_{42}b_{M\theta}+a_{43})(-a_{31}b_{V\Delta}-a_{32}b_{V\theta}+a_{34})\end{aligned} \tag{46}$$

a quantity which depends on frequency and which should vanish. Thus one, in effect, repectedly evaluates $\Delta(\omega)$ for different values of ω and finds the values of ω for which $\Delta(\omega) = 0$.

Actually it is more convenient to take the parameter

$$\chi = \beta^4 = \mu\omega^2\ell^4/EJ \tag{47}$$

as a variable rather than ω itself. Also, from (43f) and (43g) we have

$$\tau = \chi(i/\ell)^2, \quad \sigma = 2.6\tau\kappa \tag{48a,b}$$

using Poisson's ratio = 0.3. We are dealing with a circular section for which $i = a/2$. It is convenient to represent the length ℓ in terms of radius a by introducing the aspect ratio

$$n = a/\ell \tag{49}$$

Cowper (12), (13) has pointed out that Timoshenko beam theory, which is the basis of the results given in (22), employs a questionable value for the shear coefficient κ, using the value $\kappa = 4/3$ based on the elementary VQ/Ib theory. Cowper derives the value

$$\kappa = (7+6\nu)/(6+6\nu) = 1.128, \quad (\nu.3) \tag{50}$$

Brock (1), (2) and North and Roy (19) obtain the result

$$\kappa = (7+14\nu+8\nu^2)/6(1+\nu)^2 = 1.176, \quad (\nu=.3) \tag{51}$$

Cowper (12), (13) discusses the genesis of various values and Brock (3), (4) also makes such comparisons. (A source of possible confusion is that Cowper's value for a rectangle is $\kappa = (12+11\nu)/(10+10\nu) = 1.177$ which is easily incorrectly identified with the value given in Equation (51); in fact, Carnegie and Thomas (8) report Cowper's rectangle value as 1.176)

Briefly, most studies of the effect of shear deflection via Timoshenko's beam theory use $\kappa = 4/3$, but almost certainly a smaller value, about 1.13 to 1.18, should be used. In calculations, shown later, we use the common value, 4/3, and also the "better" value, 1.128.

10. Computer Program and Results

Based upon the analysis of the preceding section a computer program (see listing in Table C-1) was employed to calculate a frequency reduction factor applicable to uniform cylindrical cantilevers. This factor, designated as FRF is the ratio

$$\mathrm{FRF} = \frac{\text{lowest frequency considering effects}}{\text{lowest frequency using elementary theory}} \tag{52}$$

Calculations were made for the following added effects

1. Shear deflection only, $\kappa = 4/3$

2. Shear deflection only, $\kappa = 1.176$

3. Rotatory inertia only

4. Foundation compliance $b_{M\theta}$ only

5. Foundation compliance $b_{V\theta}$ only

6. Shear deflection ($\kappa = 1.176$) and rotatory inertia

7. Foundation compliance, $b_{M\theta}$ and $b_{V\Delta}$

8. Shear deflection ($\kappa = 1.176$), rotatory inertia, and foundation compliance, $b_{M\theta}$ and $b_{V\Delta}$.

Figure C.3 shows graphs of FRF for these conditions plotted against the ratio $B/\ell = 2a/\ell$ from $B/\ell = 0$ (ideally slender beam) up to $B/\ell = 1$ (very stubby beam).

Also shown on Figure C.3 is a ninth curve which represents pure conjecture for the additional effect of foundation inertia and foundation compliances $b_{V\theta}$ and $b_{M\Delta}$. Inasmuch as most of the non-elementary effects are represented in curve 8, the degree of conjecture in curve number 9 is not particularly great. It is hoped and expected that the FEM analysis described in Section 2 of this experiment will supply a reliable basis for establishing the curve No. 9.

We have verified the accuracy of the evaluations indicated in Fig. C.3 by comparing with the work of Gains and Volterra (14) who consider shear deflection and rotatory inertia in determining frequencies for tapered cantilevers, one of their cases being that of a uniform cylinder. Conway and Dubil (10) and Conway, Becker, and Dubil (9) have also treated the problem of conical bars but do not account for shear deflection and rotatory inertia. They dealt with an equation admitting solutions in

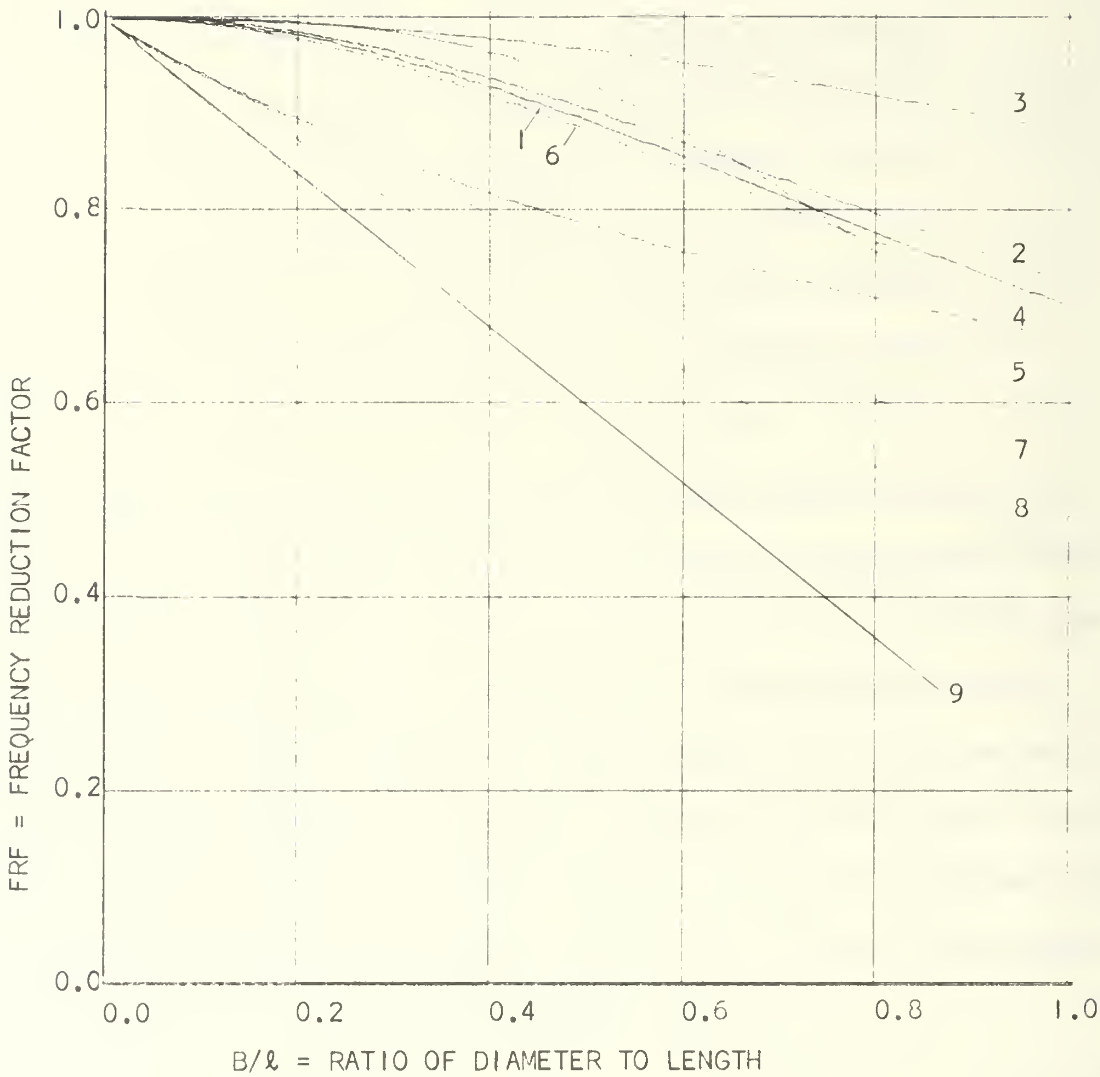


Figure C-3. FREQUENCY REDUCTION FACTOR (FRF) AS A FUNCTION OF THE RATIO OF DIAMETER TO LENGTH OF A UNIFORM, CYLINDRICAL CANTILEVER BEAM. (Numbers on curves refer to the descriptions appearing in the text; curve 9 is conjectured.

TABLE C-I

Computer program determining effects of shear deflection, rotatory inertia, and foundation compliance

```
C     J. E. BRCCK, 16 AUGUST 1974
C     PROGRAM FOR EVALUATING THE EFFECT OF SHEAR DEFLECTION, ROTATORY
C     INERTIA, AND FOUNDATION COMPLIANCE UPON THE LOWEST FREQUENCY
C     OF VIBRATION OF A UNIFORM CANTILEVER OF CIRCULAR CROSS SECTION.
C     AN EARLIER VERSION IS DATED DECEMBER 1972.
        IMPLICIT REAL*8(A-H,K,L,O-X)
        REAL*4 Y(200), Z(200)
        REAL*8 TI(12)/'BROCK, J','. E., BJ','X 3 --  ','SOFT CAN','TILEVER
       1S','   --   ','6"'                /
        REAL JAB(8)/'    1','    2','    3','    4','    5','    6','    7','    8
       1'/
        EPS=1.D-6
        NB=100
        ENB=NB
        D=1.D0/ENB
        DEL=1.D-1
        DO 20 J=1,8
        B=0.D0
        BETA=2.D0
        PRINT 6,J
      6 FORMAT(10X,'J EQUALS ',I4)
        PRINT 7
      7 FORMAT(10X,'B',7X,'BETA')
        C=0.D0
        E=0.D0
        F=0.D0
        G=0.D0
        FAC1=3.08972BD-1
        FAC2=1.50764D-1
        JJ=2
        IF(J.EQ.1)  JJ=1
        IF(J.EQ.1)  C=1.3333333333D0
        IF(J.EQ.2)  C=1.176D+0
        IF(J.EQ.3)  E=1.D0
        IF(J.EQ.4)  F=FAC1
        IF(J.EQ.5)  G=FAC2
        IF(J.EQ.6)  C=1.176D+0
        IF(J.EQ.6)  E=1.D0
        IF(J.EQ.7)  F=FAC1
        IF(J.EQ.7)G=FAC2
        IF(J.EQ.8)  C=1.176D+0
        IF(J.EQ.8)E=1.D0
        IF(J.EQ.8)  F=FAC1
        IF(J.EQ.8)  G=FAC2
        IF(J.EQ.9)  JJ=3
        DO 10 I=1,NB
        DEBET=DEL
        B=B+D
        A=BETA**4
        CALL FREAK(A,B,C,DD,E,F,G)
        D1=DD
      1 BETA=BETA-DEBET
        A=BETA**4
        CALL FREAK(A,B,C,DD,E,F,G)
        D2=DD
        S=D1*D2
        IF(S)3,4,2
      2 D1=D2
        GO TO 1
      3 IF(DEBET.LE.EPS) GO TO 4
        BETA=BETA+DEBET
        DEBET=1.D-1*DEBET
        GO TO 1
      4 CONTINUE
        OMRAT=(BETA/1.875104D0)**2
        PRINT 5,B,BETA,OMRAT
      5 FORMAT(5X,3F10.5)
        Y(I)=B
        Z(I)=OMRAT
     10 CONTINUE
        CALL DRAW(NB,Y,Z,JJ,0,JAB(J),TI,0,0,0,0,0,0,8,8,1,JJJ)
     20 CONTINUE
        STOP
```

```
        SUBROUTINE FREAK(A,B,C,D,E,F,G)
C          PARAMETER LIST:
C             A= BETA TO FOURTH POWER
C             B = DIAMETER OVER LENGTH RATIO
C             C = SHEAR FACTOR( C = 1.33333 OR C = 1.176 )
C             D = FREQUENCY DETERMINANT (OUTPUT)
C             E = ROTARY INERTIA FACTOR ( E = 1.0 OR E = 0.0 )
C             F = FOUNDATION FACTOR (ZERO OF BEE-SUB-EM-THETA)
C             G = FOUNDATION FACTOR (ZERO OR BEE-SUB-VEE-DELTA)
        IMPLICIT REAL*8(A-H,K,L,O-X)
        TAU=A*(B/4.D0)**2
        SIGMA=2.6D0*TAU*C
        TAU=TAU*E
        TEM1=DSQRT(A+(SIGMA-TAU)**2/4.D0)
        TEM2=(SIGMA+TAU)/2.D0
        L1=DSQRT(TEM1-TEM2)
        L2=DSQRT(TEM1+TEM2)
        L=1.D0/(L1**2+L2**2)
        SH=DSINH(L1)
        CH=DCOSH(L1)
        ST=DSIN(L2)
        CT=DCOS(L2)
        C0=L*(CH*L2**2+CT*L1**2)
        C1=L*(SH*L2**2/L1+ST*L1**2/L2)
        C2=L*(CH-CT)
        C3=L*(SH/L1-ST/L2)
        C4=C0-TAU*C2+F*B*(C3*(A+TAU**2)-TAU*C1)
        C5=C0-SIGMA*C2-G*A*B**3*(C1-SIGMA*C3)
        C6=A*(C3+F*B*C2)
        C7=C1-(SIGMA+TAU)*C3-G*A*B**3*C2
        D=C4*C5-C6*C7
        RETURN
        END
```

terms of Bessel functions. Carnegie and Thomas (8) consider the effects shear deformation and rotatory inertia on tapered cantilever frequencies. They provide an extension bibliography. See also Hurty and Rubinstein (15). However, none of these analyses considers the effect of foundation compliance and inertia.

11. Frequency Reduction Factors

We have shown here that foundation effects are of the same order of significance as are those due to shear deflection and rotatory inertia. (Actually our own evaluations underestimate the effect, which leads us to curve number 9 in Fig. C.3). Accordingly, evaluations in the literature necessarily provide frequency estimates which are too high. Results from FEM analysis should be most reliable but they are not available yet. Although the effect of the central cavity of a thermowell upon its vibration frequency is probably small (and probably such as to raise the frequency so that neglecting its effect is conservative) so that the data of Gaines and Volterra (14) might appear tempting, foundation effects are not considered in their data. The analysis in this Appendix is deficient in that it considers only uniform (i.e., nontapered) beams and only part of the foundation effects. A more elaborate analysis, outlined in Section 8 hereof is capable of dealing with nonuniform beams but it has not actually been programmed.

Accordingly, it would appear that the best one could do for a nonuniform beam would be either

(a) to calculate lowest frequency using elementary theory, and by assuming that the "other" effects are about the same as for a uniform beam, apply a frequency reducing factor, such as that given by curve 9 of Fig. C.3.

(b) to determine lowest frequency, including shear and rotatory inertia, using the methods or the data given by Gaines and Volterra (14) or by Carnegie and Thomas (8), and apply a (different) frequency reducing factor, based on uniform beam calculations as in the preceding section hereof, to account for foundation effects.

The aspect ratio parameter ρ in Figure C.3 must be generalized to accommodate to tapered beams and we suggest the definition

$$\rho = (\text{tip diameter} + \text{root diameter})/2 \times \text{length} \qquad (53)$$

Then, to obtain frequency ω use either formula (54) or formula (55)

$$\omega = FRF_1 \times \omega_{ELEM} = (1-.8\rho)\omega_{ELEM} \qquad (54)$$

$$\omega = FRF_2 \times \omega_{SDRI} = (1-.4\rho)\omega_{SDRI} \qquad (55)$$

where ω_{ELEM} is the value obtained by elementary analysis using a procedure like Stodola's or Rayleigh's, and where ω_{SDRI} is the value obtained (somehow) taking into account shear deflection and rotatory inertia. Neither ω_{ELEM} nor ω_{SDRI} should attempt to include foundation effects. Both FRF_1 and FRF_2 are based on our analysis here of a uniform circular beam.

We feel that formula (54) is fully as reliable as is formula (55) and it is easier to use. The two formulas agree at $\rho = 0$ and, for a uniform beam, at $\rho = 1/3$.

12. Elementary Analysis for Actual Thermowell Geometry

Using Stodola's method and a digital computer program not shown here, calculations were made for "elementary" lowest frequency for the shape shown in Figure C.4. Results obtained for a variety of parameter ratios lead to the equation

$$f_{ne} = (F_f\, A/L^2)\sqrt{E/(\gamma+\gamma')} \qquad (56)$$

where f_{ne} is the natural elementary frequency in Hertz. (i.e., cycles per second). Here A and L are dimensions as shown, E is Young's modulus and γ, γ' are specific weights, pounds per cubic inch, of thermowell material and immersing fluid respectively. A very slight error is involved in that the fluid "added" mass does not include, as it should, the mass corresponding to the volume of the central cavity.

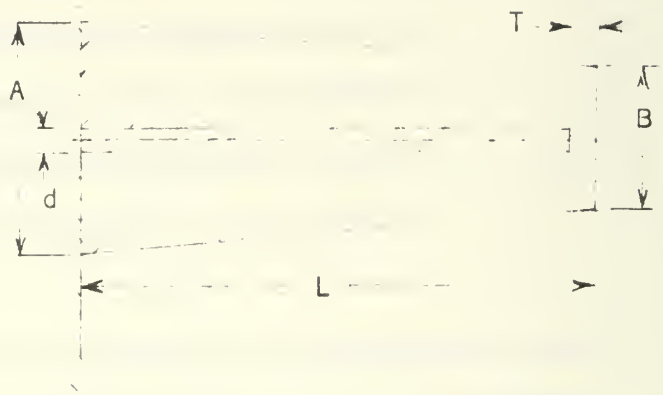


Fig. C.4 Thermowell dimensions.

Values of the factor F_f are shown in Figure C.5 as functions of the ratios B/A and d/A. A conservative lower bound is for d/A = 0 and this curve can be adequately approximated by the formula

$$F_f = 1.65 + 1.21\ (A/B)(1-.094A/B) \qquad (57)$$

13. Example of Frequency Calculations; Recommendations

Consider the case of a thermowell of P22 material for which A = 1.25", B = .625", L = 3.10", d = .25", T = .25" operating with steam at 995°F, 2350 psig. The material specific weight is $\gamma = 0.283$ pounds per cubic inch. The fluid specific volume is 0.328 cu. ft per pound, which gives $\gamma' = .0018$ say .002 pounds per cubic inch. At this temperature E = 23,100,000 psi. We also calculate B/A = .5, d/A = .2 and from Fig. C.5 get $F_f = 3.83$. Thus the elementary value of natural frequency is

$$f_{ne} = \frac{(3.83)(1.25)}{(3.10)^2} \sqrt{\frac{23100000}{.285}} = 4485 \text{ Hz.}$$

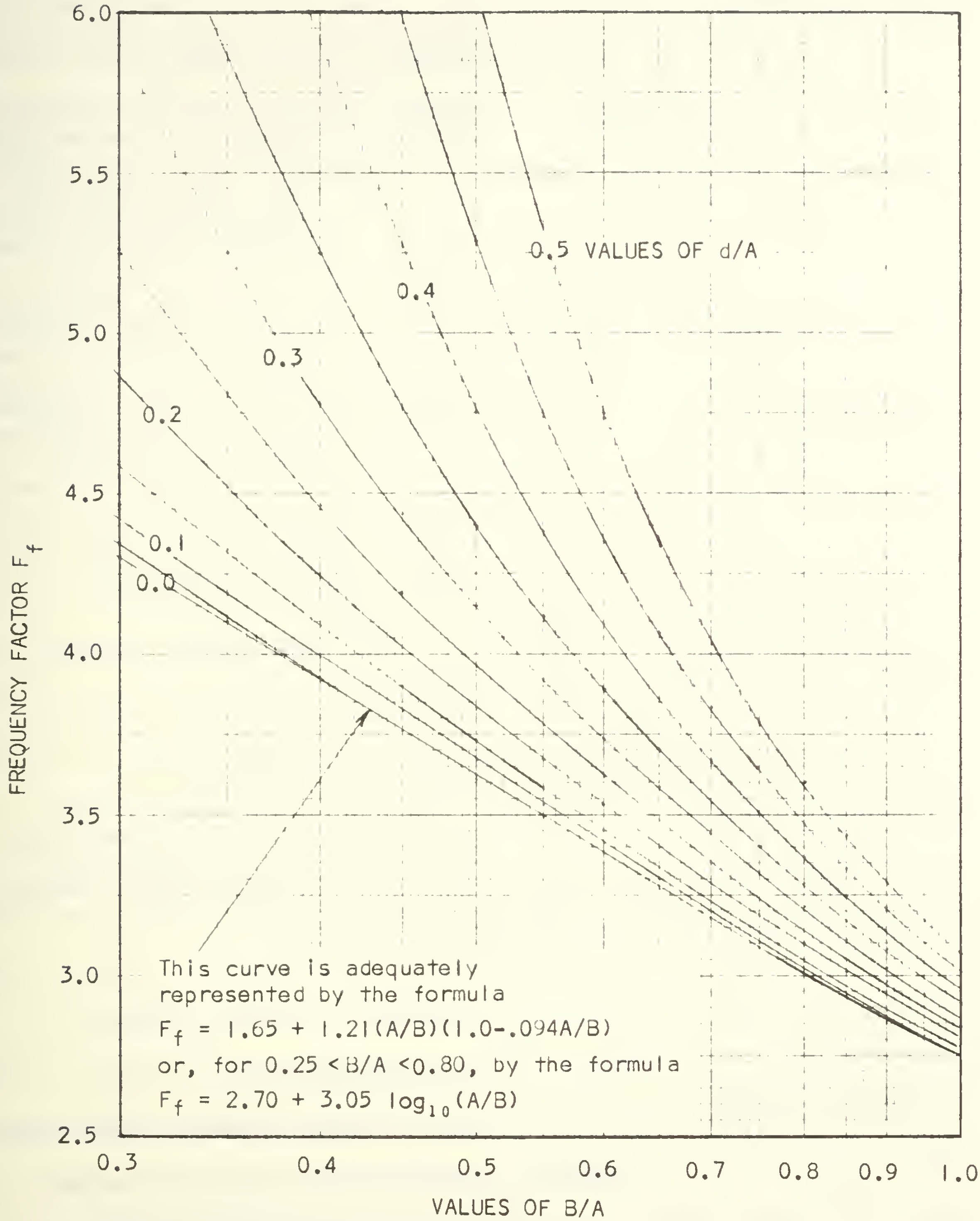


Figure C-5 Values of the frequency factor F_f as a function of the ratio B/A for various values of d/A. All curves are for T/L = 0. Nonzero values of T/L result in very slight reduction of the values shown here.

$$FRF_1 = 1-(.8)(1.25+.625)/6.20 = .758$$

Thus we estimate

$$f = 3400 \text{ Hz}$$

If we had used the lower bound given by Equation (57), we would have calculated $F_f = 3.62$, $f_{ne} = 4239$ Hz, $f = 3213$ Hz which is conservative.

The alternate procedure would employ, for example, the data of Gaines and Volterra (14), neglecting the central hole. Using their notations, $\delta = .5$ (fortunate! since there is thus no need to interpolate on δ), and $\lambda = 2.48$. This value, however is beyond the range of the data they give. If we are to proceed by this route, we would extropolate their data. An approximate value is $\Phi = 4.30$, whence, using their analysis

$$f = (4.30A/2\pi L^2)\ \sqrt{24.1E/(\gamma+\gamma^1)} = 3.36A/L^2\sqrt{E/(\gamma+\gamma^1)} = 3935 \text{ Hz}$$

This value, 3.36, compares to our value 3.62 obtained above. Next, accounting for the foundation effects, we get

$$f = [1-(.4)(1.25+.625)/6.20](3935) = 3460 \text{ Hz}$$

which compares with our value $f = 3213$ Hz given above. These comparable values are too low because of the effect of the central cavity. Assuming the same correction applies, we finally calculate

$$f = (3.83)(3460)/(3.62) = 3660 \text{ Hz}$$

which compares with our earlier evaluation, 3400 Hz.

For the purpose at hand, our simple procedure (Equation (54)) gives 3400 Hz and the alternate procedure (Equation (55)) gives 3660 Hz. We simply do not know which is more accurate, but for our purposes 3400 Hz is more conservative (i.e., safer) and it is surely easier to obtain.

Thus, we recommend the simpler procedure. We outline the procedure in Appendices F and G, where, it should be noted, that the lower bound formula (57) might be used first to estimate F_f. If results are not satisfactory, a more elaborate estimate of F_f may be obtained by the use of the curves given in Figure C.5 or by appropriate calculations.

14. Pipe Ovalization Mode

With one exception, studies of thermowell vibrations have dealt exclusively with what might be called cantilever modes. Interest was focussed on assuring that the lowest cantilever mode was slower than the excitation frequency. However, in an earlier report (5), (6), the writer called attention to the fact that the vortex shedding excitation could excite a "pipe ovalization mode" having a frequency slower than that of the lowest cantilever mode. This subsection discusses the pipe ovalization mode and provides assurance that no concern need be felt about it.

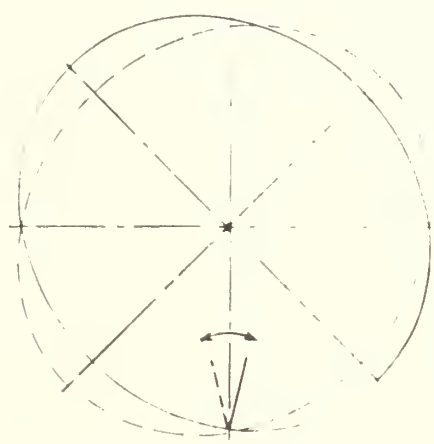


Fig. C.6 Pipe ovalization mode.

A standard treatment of nonextensional ovalizing vibrations for thin circular rings is given by Timoshenko (23). Rewriting Timoshenko's formula so as to apply to thin circular cylinders, and taking the number of "lobes" to be equal to 2 so as to obtain the lowest (ovalizing) mode, we get

$$\omega^2 = 7.2EIg/A\gamma r^4(1-\nu^2) \tag{58}$$

where r = mean radius of cylindrical shell

E = Young's modulus (30,000,000 psi for steel)

ν = Poisson's ratio (0.3 for steel)

I = moment of inertia of unit strip of pipe wall about an axis passing through mid-thickness ($I \approx t^3/12$)

g = 386 in/sec^2

A = area of unit strip = t

γ = specific weight (.283 for steel)

t = wall thickness

There is some added or virtual fluid mass which also participates in the motion so that γ should be slightly increased to account for this behavior. Our tubes are thick, not thin. Accordingly, the virtual fluid mass is so much smaller than the metal mass that we will not concern ourselves further with this correction.

Thus, for steel tubes we find

$$\omega = 164300t/r^2; \quad f = 26142t/r^2 \text{ (Hertz)} \tag{59}$$

where t and r are measured in inches. The formula is really applicable only to thin tubes and the pipes (such as main steam lines) with which we are concerned are hardly thin. Accordingly there are other compliances in action that serve to reduce the frequency below that given by Equation (59). Nevertheless, this is about as well as we are able to do; furthermore this estimate will be satisfactory for our purposes.

This formula gives astonishingly low values for the ovalizing frequency f. Thus, in the case of a main steam pipe with t = 3", r = 10", say, we find f = 784 Hz.

This is almost certainly less than the excitation frequency in a main steam system. (Actually we might be concerned with three-lobe ovalization for which the frequency is 2.83 times as great as that given by Equation (59).

Then we see that there are structural frequencies, slower than the lowest cantilever frequency, with which the vortex shedding excitation could couple. The obvious question is whether or not we should be concerned with such case. A plausible argument that we need feel no concern is offered below. However, it would be of the greatest interest to see whether such ovalizing modes might be observed in practice.

Figure C.6 indicates that alternate vortex shedding produces fluid forces on the thermowell which, in turn, produce moments at the root which couple with and could excite the flexural oscillations. However, note that the thermowell root is a node of the ovalized vibrations. In other words, the circumferential bending moment is zero at this point (and its opposite) and is extremal at the 45° positions. If the problem were truly two dimensional (plane strain) and if there were no energy removal, the points subject to maximum stress, and thus the points most susceptible to fatigue failure,would lie along longitudinal lines at the 45° positions. The stresses at the root of the cantilever would be essentially zero; certainly much less than would occur for what we have called cantilever modes.

Furthermore, this two dimensional presentation is an oversimplification. Theoretically, the vortex shedding from the thermowell, which is located at a definite axial position along the pipe, could excite ovalized vibrations that would extend, without change, to infinity in both directions. However, the pipe itself doesn't and there are energy sinks upstream and downstream which prevent a build-up of distortion such as illustrated in Figure C.6. Locally there is energy absorbtion by fluid contents and external insulation or other coatings; however, it has been shown experimentally that these mechanisms do not afford much damping capability (18).

The above remarks apply not only to the two-lobe case shown in Figure C.6 but, generally also to ovalizations with more than one lobe.

Briefly our reason for advocating that we forget about these pipe-ovalizing vibrations when considering thermowell integrity is that they do not imply significant stress in the thermowell. The greatest stress associated with ovalization is in the pipe wall itself, at 45° from the thermowell root (for the two-lobe case) and there seems to be no experimental evidence that the pipe is thereby itself endangered.

15. Bibliography for Appendix C

C1. Brock, J. E., Shear distribution in piping, Heating Piping and Air Conditioning, Vol. 35, No. 1, January 1963, pp. 141-143.

C2. Brock, J. E., Memorandum A, Remarks on the shear distribution factor in piping, Report to U. S. Navy Marine Engineering Laboratory under Contract N161-26147, June 21, 1965.

C3. Brock, J. E. Appendix B to Fifth Progress Report, November 15, 1966, to United States Naval Academy, MEL No. 72-117, under Contract N161-26327.

C4. Brock, J. E. Appendix A to Sixth Progress Report, January 15, 1967, to United States Naval Academy, MEL No. 72-117, under Contract N161-26327.

C5. Brock, J. E., Stress analysis of thermowells, consulting report to Bechtel Power Corporation, October 1972.

C6. Brock, J. E., Supplement to Stress analysis of thermowells, consulting report to Bechtel Power Corporation, December 15, 1972.

C7. Brown, J. M., and Hall, A. S., Bending deflection of a circular shaft terminating in a semi-infinite body, J. Appl, Mech., Vol. 29, No. 1, March 1962, pp. 86-90.

C8. Carnegie, W., and Thomas, J., The effects of shear deformation and rotary inertia on the lateral frequencies of cantilever beams in bending, J. Eng'g. Ind., Vol.— , No. 1, February 1972, pp. 267-278. (Trans, ASME, Ser. B, Vol. 94, No. 2) February 1972, pp. 267-278.

C9. Conway, H. D., Becker, E. C. H., and Dubil, J. F., Vibration frequencies of tapered bars and circular plates, J. Appl. Mech., Vol. 31, No. 2, June 1964, pp. 329-331.

C10. Conway, H. D., and Dubil, J. F., Vibration frequencies of truncated-cone and wedge beams, J. Appl. Mech., Vol 32, No. 4, December 1965, pp. 932-934.

C11. Cook, R. D., Deflections of a series of cantilevers due to elasticity of support, J. Appl. Mech., Vol. 34, No. 3, September 1967, pp. 760-761.

C12. Cowper, R., The shear coefficient in Timoshenko's beam theory, J. Appl. Mech., Vol. 33, No. 2, June 1966, pp. 335-340.

C13. Cowper, R., On the accuracy of Timoshenko's beam theory, J. Eng'g. Mech. Div., Proc. ASCE (EM 6) December 1968, pp. 1447-1453.

C14. Gaines, J. H., and Volterra, E., Upper and Lower bounds of frequencies for cantilever bars of variable cross section, J. Appl. Mech., Vol. 33, No. 4, December 1966, pp. 948-949.

C15. Hurty, W. C., and Rubinstein, M. F., On the effect of rotatory inertia and shear in beam vibration, J. Frank. Inst., Vol. 278, No. 2, August 1964, pp. 124-132.

C16. Love, A. E. H., Elasticity, 4th Ed., Dover, 1944.

C17. Matusz, J. M., O'Donnell, W. J., and Erdlac, R. J., Local flexibility coefficients for the built-in ends of beams and plates, J. Eng'g Ind., (Trans, ASME, Ser. B, Vol. 91, No. 3), August 1969, pp. 607-614.

C18. McGarrah, W. E., and Brock, J. E., Designers can predict fundamental vibration frequencies in piping structures, Heating Piping and Air Conditioning, Vol. 33, No. 8, August 1961, pp. 105-109.

C19. North, E. L., and Roy, P. A., Investigation of stress analysis practices for submarine main sea water piping, GD/EB Report U411-65-001 to MEL under Contract N161-25962, January 1965.

C20. O'Donnell, W. J., The additional deflection of a cantilever due to the elasticity of the support, J. Appl. Mech., Vol. 27, No. 3, September 1960, pp. 461-464.

C21. O'Donnell, W. J., Stresses and deflections in built-up beams, J. Eng'g. Ind. (Trans. ASME, Ser. B, Vol. 85, No. 3), August 1963, pp. 265-273.

C22. Pestel, E. C., and Leckie, F. A., Matrix methods in Elastomechanics, McGraw-Hill, 1963.

C23. Timoshenko, S. P., Vibration problems in engineering, 2nd Ed., D. Van Nostrand Co., Inc., 1937.

C24. Timoshenko, S. P., History of Strength of Materials, McGraw-Hill, 1953.

C25. Westergaard, H. M., Theory of Elasticity and Plasticity, Dover, 1964.

APPENDIX D

BENDING STRESSES AND THE COMBINED STRESS CRITERION

TABLE OF CONTENTS

1. Introduction

This Appendix deals with the bending stress that result from lateral loads imposed on the thermowell by the flowing fluid. Figure D.1 shows the configuration which will be analyzed. Note that there is allowance for a shielded length αL (where $0 \leqq \alpha < 1$) which is not exposed to the action of the fluid stream.

In the writer's earlier analysis (1), no provision was made for such shielding; i.e., $\alpha = 0$ was implicit in the analysis. Also, it was tacitly assumed that the greatest bending stress occurred at the root. As will be seen in what follows, this assumption is correct (for $\alpha = 0$). However, an algebraic error was made in the analysis as reported in (1). Subsequently, when considering the effect of non-zero shielding, the writer detected his earlier error, but made a second error, so that a notice sent to members of the ASME Committee PB51 (2) was itself incorrect.

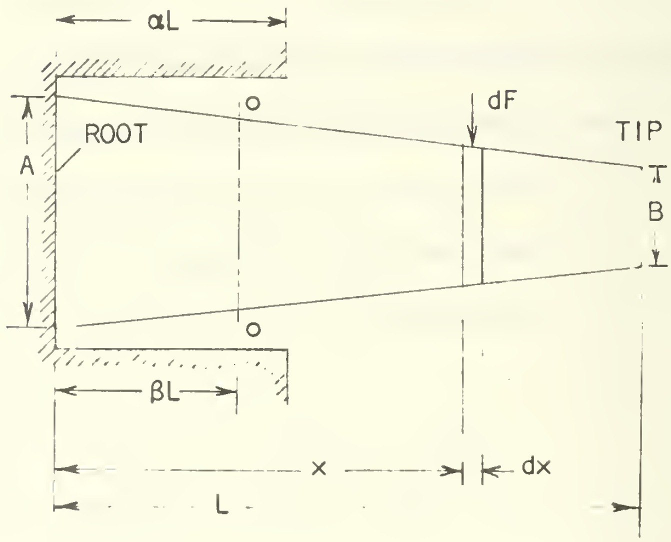


Fig. D.1 Figure for analysis of bending stresses.

2. Moment and Bending Stress Analysis

From the results reported in Appendix B it may be seen that the elemental force dF shown in Figure D.1 is

$$dF = (C\rho U^2/2)(D)(dx) \qquad (1)$$

where D is the local diameter. The coefficient $C = C_L$ if we are dealing with the lift force whereas $C = C_D$ if we are dealing with the drag force. Introducing the useful parameter

$$p = (A-B)/A \tag{2}$$

we have

$$D = A(1-px/L) \tag{3}$$

so that the bending moment at section 0 - 0 is

$$M_\beta = (AC\rho U^2/2) \int_{\eta L}^{L} (x-\beta L)(1-px/L)dx \tag{4}$$

where η is the larger of α and β, i.e.,

$$\eta = \text{Max}\ \{\alpha, \beta\} \tag{5}$$

Performing the integration gives

$$M_\beta = (AC\rho U^2L^2/12)[3(1+p\beta)(1-\eta^2)-6\beta(1-\eta)-2p(1-\eta^3)] \tag{6}$$

Obviously M_β is greatest at the root and decreases steadily toward the tip. However, the section modulus

$$Z = \pi D^3/32 \tag{7}$$

also decreases steadily from root to tip. Accordingly it is necessary to be adroit in examining the maximum of the bending stress

$$\sigma_\beta = M_\beta/Z = GH \tag{8}$$

where

$$G = 8C\rho U^2L^2/3\pi A^2 \tag{9}$$

and

$$\begin{aligned} H &= H(p,\ \eta,\ \beta) \\ &= [3(1+p\beta)(1-\eta^2) - 6\beta(1-\eta)-2p(1-\eta^3)]/(1-p\beta)^3 \end{aligned} \tag{10}$$

The value of coefficient H depends on the tapering parameter p, the location parameter β, and the parameter η. For given α, if $\beta < \alpha$ then we must take $\eta = \alpha$ while if $\beta > \alpha$ then we must take $\eta = \beta$. It takes considerable algebra to show that

$$\frac{d}{d\beta} H(p, \beta, \beta) = 0 \tag{11}$$

only at $\beta = 1$ (i.e., the tip) while

$$\frac{d}{d\beta} H(p, \alpha, \beta) = 0 \tag{12}$$

only at

$$\beta = \beta^* = \frac{p(2-p)(1+\alpha) - p^2\alpha^2-1}{(2-p-p\alpha)p} \tag{13}$$

The value of β^* may be negative, which is meaningless; in this case the maximum is at the root, $\beta = 0$. The analytical expression for $H(p, \alpha, \beta^*)$ appears to be too complicated for convenient use; it seems to be preferable to compute β^* from Equation (13) and substitute into the formula for $H(p, \alpha, \beta^*)$.

A set of curves of the parameter H, for fixed p = 0.5 (corresponding, say, to A = 1.5", B = .75") and for various shielding ratios α is shown in Figure D.2. We are essentially interested only in the maximum value of H. Accordingly, a set of curves of H_{max} as a function of p for various values of α is shown in Figure D.3.

It is only for large values of p and/or α that the maximum stress occurs other than at the root. We can find the critical relation by equating $\beta^* = 0$, cf. Equation (13). This yields

$$\alpha = (2-p-\sqrt{4p-3p^2})/2p \tag{14}$$

and a plot of this relationship is shown in Figure D.4

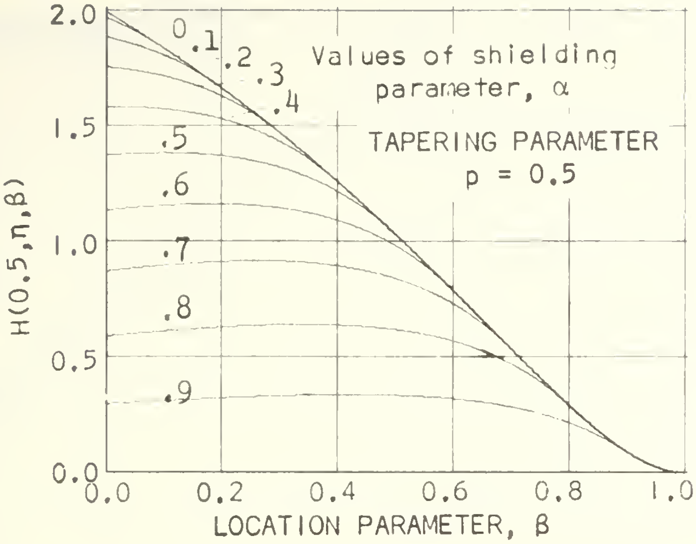


Figure D-2 The function H(0.5,n,β).
Cf. equations (5) and (10)

Table D-1 Computer program for drawing
Figure D-2, immediately above.

```
C     PROGRAM WRITTEN 29 AUGUST 1974 BY J. E. BROCK
C     TO DRAW FIGURE D-2 WITH P = 0.5
      IMPLICIT REAL*8(A-W)
      REAL*4 X(802),Y(802)
      REAL*8 TI(12)/'BROCK, J','. E., BO','X 8 --  ','BENDING ','STRESS
     1F','ACTOR   ', 6*'        '/
      REAL LA/'    '/
      INTEGER I,J,K,IN,JN,JOUT,JSKIP
      F(B)=(3.D+0*(C+P*B)*(D-ETA**2)-6.D+0*B*(D-ETA)-2.D+0*P*(D-ETA**3))
     1/(C-P*B)**3
      C=1.C+0
      P=5.D-1
      JN=801
      JSKIP=40
      EJN=JN
      IN=11
      DELTAB=D/(EJN-D)
      DO 100 I=1,IN
      EI=I-1
      ALFA=EI/(1.C+1)
      DO 20 J=1,JN
      EJ=J-1
      BETA=EJ*DELTAB
      ETA=DMAX1(ALFA,BETA)
      EFF=F(BETA)
      Y(J)=EFF
      X(J)=BETA
      IF(MOD(J,JSKIP).EQ.1) PRINT 2,P,ALFA,BETA,EFF
    2 FORMAT(5X,4F10.5)
   20 CONTINUE
      IF(I.EQ.1) K=1
      IF(I.NE.1)K=2
      IF(I.EQ.IN)K=3
      CALL DRAW(JN,X,Y,K,0,LA,TI,0,0,0,0,0,0,8,8,1,JOUT)
      BSTAR=((D+D-P)*(D+ALFA)-P*ALFA**2-D/P)/(D+D-P-P*ALFA)
      ETA=ALFA
      EFFB=F(BSTAR)
      EFFO=F(0)
      PRINT 3,BSTAR,EFFB,EFFO
    3 FORMAT(8X,3F10.5)
  100 CONTINUE
      STOP
      END
```

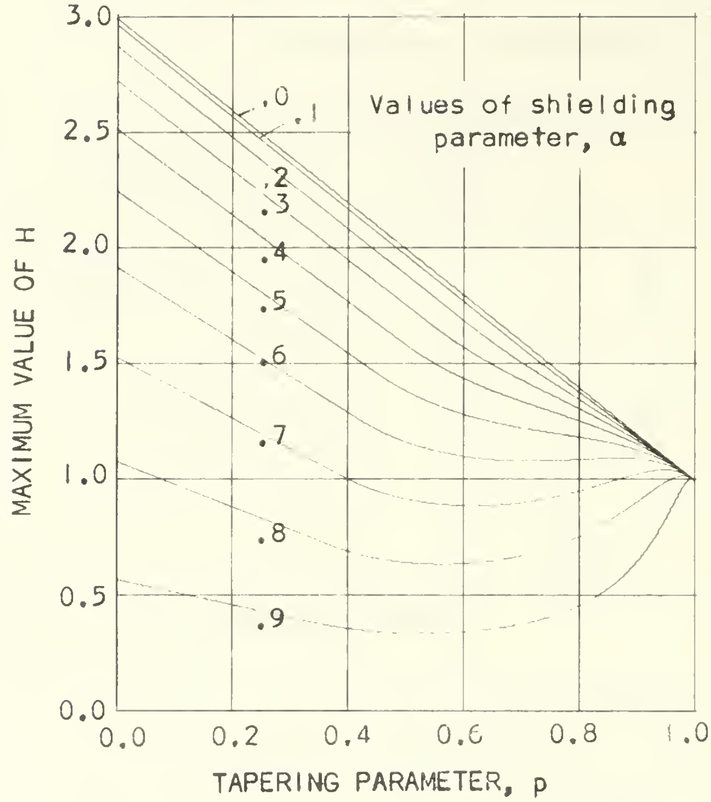


Figure D-3 Maximum of the function H(p,η,β) as a function of the tapering parameter p for various values of the shielding parameter α.

Table D-2 Computer program for drawing Figure D-3

```
C      PROGRAM WRITTEN 30 AUGUST 1974 BY J. E. BROCK TO DRAW FIG. D-3
       IMPLICIT REAL*8(A-W)
       REAL*4 X(801),Y(801)
       REAL*8 TI(12)/'BROCK, J','. E., BO','X 8 --  ','MAXIMUM ','STRESS
      1F','ACTOR',6*'        '/
       REAL LA/'        '/
       INTEGER NP,NA,I,J,K,JOUT,NSKIP
       H(P,E,B)=(3.D+0*(O+P*B)*(O-E*E)-6.D+0*B*(O-E)-2.D+0*P*(O-E**3))/
      1(C-P*B)**3
       BS(P,A)=(P*(C+C-P)*(C+A)-(P*A)**2-O)/((O+O-P-P*A)*P)
       C=1.D+0
       NSKIP=80
       NP=800
       ENP=NP
       NA=10
       ENA=NA
       DO 100 I=1,NA
       EIM=I-1
       A=EIM/ENA
       DO 50 J=1,NP
       EJ=J
       P=EJ/ENP
       X(J)=P
       B=DMAX1(0.D+0,BS(P,A))
       E=DMAX1(A,B)
       Y(J)=H(P,E,B)
       IF(MOD(J,NSKIP).EQ.0) PRINT 20,A,B,X(J),Y(J)
 20    FORMAT(5X,4F10.5)
 50    CONTINUE
       IF(I.EQ.1) K=1
       IF(I.NE.1) K=2
       IF(I.EQ.NA) K=3
       CALL DRAW(NP,X,Y,K,0,LA,TI,0,0,0,0,0,0,8,8,1,JOUT)
 100   CONTINUE
       STOP
       END
```

It is clear that for most practical cases the maximum does occur at the root. The maximum occurs at the root in all cases if $p \leqq 1/3$, a typical value, and, if the shielding ratio α does not exceed 0.5, then the maximum occurs at the root for $p \leqq .453$. Accordingly, it will be only in the rarest of cases that the maximum stress occurs other than at the root.

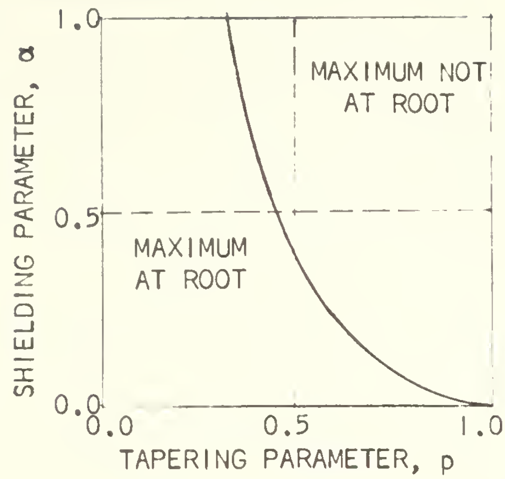


Fig. D.4 Cases where maximum bending stress is (is not) at the root.

Accordingly, the value of H which is of primary interest is

$$H = H(p, \alpha, 0) = 3(1-\alpha^2)-2p(1-\alpha^3)$$

$$= (3-2p)(1-\alpha^2)-2\alpha^2 p(1-\alpha) \qquad (15)$$

The second term above is small so that we will be making only a small and conservative error by taking

$$H = (3-2p)(1-\alpha^2) = (1-\alpha^2)(A+2B)/A \qquad (16)$$

and the bending stress becomes

$$\sigma = 8C\rho U^2 L^2 (1-\alpha^2)(A+2B)/3\pi A^3 \qquad (17)$$

When we first reported this result (1), there was no factor $(1-\alpha^2)$ since we did not contemplate shielding and we erroneously gave (5A-3B) in place of (A+2B). Later, (2) when we included the $(1-\alpha^2)$ term, we were still wrong, reporting (5A-2B) rather than (A+2B).

3. Combination of Bending Moments and Stresses

The drag force is approximately constant in time. The lift force, which is at right angles to the drag force, varies in time. The bending moments due to the lift and drag forces are at right angles and combine vectorially. The effective value of the lift force should be multiplied by a dynamic intensification factor

$$K = 1/\left|1-(f_s/f_n)^2\right| \qquad (18)$$

where f_s, f_n are the Strouhal frequency (see Appendix B) and the natural or resonant frequency (see Appendix C) respectively. The vertical lines indicate "absolute value."

In the preceding analysis we made a slight oversimplification by taking Z as given by Equation (7). This does not account for the hollow center of the thermowell. If we had taken this into account, the demarcation shown in Figure D.4 would have moved slightly to the right. However, our conclusion that practical cases involve highest stress at the root would still be valid. We should however, compute bending stress by using the correct value of Z at the root. This means that the stress values given above should be corrected by multiplying by the factor

$$Z_{WRONG}/Z_{RIGHT} = i/(1-d^4/A^4) \qquad (19)$$

where d is the diameter of the hole down the center. Thus, we obtain

$$\sigma^* = \sigma(\text{bending, max}) = 8C\rho U^2L^2(1-\alpha^2)A(A+2B)/3\pi(A^4-d^4) \qquad (20)$$

where C is the effective value

$$C = \sqrt{C_D{}^2+K^2C_L{}^2} \qquad (21)$$

Recalling from Appendix B the recommended values of C_D and C_L, we get

$$C = \sqrt{1.44 + 1.69K^2} \quad \text{for} \quad N_R < 10^5 \tag{22a}$$

$$C = \sqrt{9 + K^2}/4 \quad \text{for} \quad N_R > 10^5 \tag{22b}$$

4. Combination with Pressure Stress

We examine the conditions at the root at the inside and outside surface. Presuming elastic stress distribution, we have the following stress components to be concerned about, cf. Table D.3.

	σ_r	σ_θ	σ_{AXIAL}
OUTSIDE	$-p$	$-p(A^2+d^2)/(A^2-d^2)$	$-pA^2/(A^2-d^2) \pm \sigma^*$
INSIDE	0	$-2p^2A^2/(A^2-d^2)$	$-pA^2/(A^2-d^2) \pm \sigma^* d/A$

(23a,b,c,d)

TABLE D.3 STRESS COMPONENTS

We use the Tresca or Guest condition, as in Appendix A and as is basic in the ASME code. Considering first the inside conditions, if $\sigma^* < pA^3/d(A^2-d^2)$ which is a reasonable assumption except for quite long and slender thermowells, then σ_{AXIAL} will be the intermediate stress and the equivalent stress or so-called "stress intensity" will be

$$S_E(\text{inside}) = 2pA^2/(A^2-d^2) \tag{24}$$

The bending stress does not appear here; the stresses due to pressurization were dealt with fully in Appendix A and the criteria given there provide assurance of continuous integrity not only at the root, which we are examining here but also at the tip where the stresses are higher than at the root if A > B. Accordingly, we must consider the stress situation at the outside. If σ^* were sufficiently small, the axial stress would be intermediate and bending would not affect the integrity assured by the pressure design criteria. Accordingly we must assume that $\sigma^* > pd^2/(A^2-d^2)$ and the governing stress intensity becomes

$$S_E = \sigma^* + pd^2/(A^2-d^2) \qquad (25)$$

According to ASME criteria we must require that this value not exceed 1.5 S_m where S_m is the tabulated stress value for the material in question at design temperature.

There is one additional matter to take care of before proceeding. Usually the fluid velocity U will be given in feet per second and the density ρ will be given in pounds seconds^2/foot^4 according to the formula

$$\rho = 1/gv \qquad (26)$$

with g = 32.2 feet/second and v = specific volume in cubic feet per pound. Thus the product $\rho U^2 = U^2/gv$ will be in pounds per square foot. This must be divided by 144 to give ρU^2 in pounds per square inch as required by our formulas. Thus, we require that

$$pd^2/(A^2-d^2) + C\rho U^2L^2(1-\alpha^2)(A+2B)/54\pi A^3(1-d^4/A^4) \leqq 1.5\ S_m \qquad (27)$$

5. Combined Stress Criterion

We will rewrite this to include all applicable numerical values, viz.:

$$pd^2/(A^2-d^2) + .00018\ CU^2L^2(1-\alpha^2)(A+2B)/vA^3(1-d^4/A^4) \leqq 1.5\ S_m \qquad (28)$$

with A, B, L, and d given in inches, p and S_m given in psi, U given in feet per second, and v given in cubic feet per pound. C and α are dimensionless.

For certain purposes we can simplify this formula. For example, for high pressure installations $1-d^4/A^4 \approx 1$ which gives a small (nonconservative) error. Also, in accordance with the limitation recommended in the body of this report that f_s/f_n not be permitted to exceed 0.80, the dynamic intensification factor K will not exceed 2.78 so that we can say

$$C < 3.81 \quad \text{for} \quad N_R < 10^5 \tag{29a}$$

$$C < 1.02 \quad \text{for} \quad N_R > 10^5 \tag{29b}$$

6. Bibliography

D1. Brock, J. E., Stress analysis of thermowells, consulting report to Bechtel Power Corporation, October 1972, and Supplement, December 15, 1972.

D2. Appendix to letter J. E. Brock to J. W. Murdock, March 14, 1974 Copies sent to Mr. W. O. Hayes for ASME Committee PB51.

APPENDIX E

FATIGUE RELIABILITY CALCULATIONS

We state explicitely that we are concerned here only with cyclic response to the sinusoidally varying F_L force. (This was implicit in an earlier version of this report.) As justification we remark that stress cycles due to pressure and temperature variations are surely fewer in number than those due to F_L and, indeed, are no more severe than for the pipe itself. In particular, recall that we require one-cycle shakedown on pressurization. However, we do want to call specific attention to the fact that we have thus limited the scope of the fatigue investigation.

The number of cycles in a twenty-year design life is

$$N = (20)(8760)(3600)f_s$$

and a typical value for f_s is $.21U/D = (.21)(270)(12) = 680$ Hz so that $N = 4.3 \times 10^{11}$. Thus we are concerned with a very large number of excitations, in the range 10^{11} to 10^{12} roughly. To assure survival against the action of mechanical fatigue, we must require that the amplitude of maximum cyclic stress (including so-called "peak" stress) does not exceed the endurance limit for the material at the operating temperature. The endurance limit may be taken to be twice the ASME Code S_a value at one million cycles as obtained from the Design Fatigue Curves in Appendix 1 of the ASME Code for Nuclear Power Plant Components or in ASME Boiler Code Case No. 1331.

From equation (20) of Appendix D we have

$$\sigma = 8C\rho U^2 L^2 (1-\alpha^2) A(A+2B)/3\pi(A^4-d^4) \qquad (1)$$

for the bending stress at the root; this is the maximum value of bending stress, as explained in Appendix D, except for thermowells which are much more sharply tapered or are much more completely shielded than ususal. We are here concerned only with the effect of the lift force so that

$$C = KC_L = \begin{cases} 1.3K & \text{for} \quad N_R < 10^5 \\ .25K & \text{for} \quad N_R > 10^5 \end{cases} \tag{2}$$

where we recall that the dynamic intensification factor K is given by

$$K = 1/\left|1 - (f_s/f_n)^2\right| \tag{3}$$

(cf. equation 18 of Appendix D.)

However, equation (1) above does not account for the intensifying effect of what may be a sharply notched geometry at the root. Comparing with the procedure in nuclear codes we should multiply by the product (K_2C_2 in nuclear code notation) of the two stress indices pertinent to the local geometry. Proceding on a "rational" basis using tabulated values of K_2 and C_2 and the general philosophy of Code Case No. 69, we arrive at

$$K_2C_2 = (\frac{4}{3} \times 2)(\frac{2}{1.8} \times 1.5) = 4.4 \tag{4}$$

Since this evaluation is doubtful and until better information is available we will round this number up to $K_2C_2 = 6$.

Inserting all evaluations into (1) and multiplying by 6 we get

$$S_{alt} = \begin{Bmatrix} .00143 \\ .000275 \end{Bmatrix} \frac{KU^2L^2(1-\alpha^2)A(A+2B)}{v(A^4-d^4)} \tag{5}$$

We have denoted this result as S_{alt} to denote the alternating stress which may lead to fatigue damage. The upper figure in parentheses is for $N_R < 10^5$ and the lower figure for $N_R > 10^5$. The dimensions are: L, A, B, d (inches); U(ft/sec); V(cu. ft/lb); S_{alt} (psi); f_s, f_n(Hz).

We require that this value, S_{alt}, not exceed the endurance limit for the material at operating temperature.

If pressurization and depressurization are to take place very frequently or if significant and highly repetitive thermal transient stresses are possible one should also investigate these as a possible source of fatigue damage.

The same sort of simplifications are possible with equation 5, above, as were suggested for use with equation (28) of Appendix D.

APPENDIX F

PLAN AND SEQUENCE OF CALCULATIONS

1. Obtaining and recording data

   a. Service, identification, etc. Record appropriate information identifying the service (MS, HRH, CRN, BFW, etc.), plant, thermowell location etc.

   b. Fluid data. Record:

      p, pressure (psig)

      T, temperature (^{o}F)

      U, velocity (feet per second)

   c. Thermowell dimensions (all in inches)

      A, root diameter

      B, tip diameter

      d, hole diameter

      t_{AVG}, average tip thickness

      L, length (root to tip)

      SL, shielded length (αL in Figure D-1)

   d. Metal properties. Unless otherwise indicated assume metal temperature equals T, recorded above for the fluid. Determine and record:

      S_M, tabulated stress value (psi)

      S_{end}, endurance limit (psi)

      E, Young's modulus (psi)

      γ, specific weight (pounds per cubic inch)

   Notes: (1) S_{end} may be taken to be equal to twice the S_a value at one million cycles; cf. ASME Code for Nuclear Power Plant Components, Appendix 1 (2), or ASME Code Case No. 1331 (1).

(2) All quantities should be determined at metal operating temperature.

(3) Tables of Young's modulus may be found for example in Appendix C of the Power Piping Code (5).

(4) For most metallic materials used for thermowell construction $\gamma = .283$ pounds per cubic inch, approximately.

e. Fluid properties. From ASME Steam Tables (3) or other appropriate source determine and record:

v, specific volume (cubic feet per pound)

ν_k, kinematic viscosity (ft^2 per 1000 seconds)(used in calculating Reynolds number)

2. <u>Pressure calculation</u>

a. Calculate d/B

b. Calculate P_a

If $d/B \leqq .45$, $P_a = [1-(d/B)^2]S_M$

If $.45 \leqq d/B \leqq .6$, $P_a = -S_M \log_e(d/B)$

If $.6 \leqq d/B \leqq .8$, calculate P_{10} (see below), and then $P_a = (4-5d/B)(.51S_M) + (5d/B-3)P_{10}$

If $.8 \leqq d/B < 1$, use the rules of Par. UG-28 of Division 1, Section VIII of the ASME Boiler and Pressure Vesses Code. (4)

Note: P_{10} is the allowable pressure, under Div. 1, Section VIII of the ASME Code, for the particular value d/B = .8, corresponding to $D_o/t = 10$.

Thermowells for high pressure service will satisfy $d/B < .6$ so that there will be no need to calculate P_{10}.

c. <u>It is required that p (fluid pressure), not exceed the value of P_a calculated above</u>.

3. Tip thickness verification

Calculate $d\sqrt{p/S_M}$. It is required that $t_{AVG} \geqq d\sqrt{p/S_M}$ and $t_{min} \geqq \frac{d}{2}\sqrt{p/S_M}$.

4. Excitation Frequency Calculation

a. Calculate Reynolds number for the flow past the thermowell. $N_R = 83UA/\nu_k$ where

A = thermowell root diameter (inches), U = fluid velocity (feet per second), ν_k = (ASME tabulated) kinematic viscosity (ft^2 per 1000 seconds)

b. Get Strouhal number N_S

For $N_R < 4 \times 10^4$, take $N_S = .21$

For $4 \times 10^4 < N_R < 4 \times 10^5$, take $N_S = .24 \log_{10} N_R - .894$

For $N_R > 4 \times 10^5$, take $N_S = .45$

c. Calculate Strouhal frequency

$$f_s = 12 N_s U/A$$

(Note that N_R and f_s are based on the thermowell diameter, specifically the root diameter, and not on the inside diameter of the pipe.)

5. Cantilever response frequency calculation

a. Determine the frequency factor F_f from Figure C-5 (Appendix C), or use the conservative formula $F_f = 1.65 + 1.21\,(A/B)(1.0 - .094A/B)$

b. Calculate the elementary value of natural frequency f_{ne}

$$f_{ne} = (F_f\, A/L^2)\,\sqrt{E/(\gamma+\gamma')}$$

Here γ' = fluid specific weight, pounds per cubic inch = .00058/v

c. Calculate the frequency reduction factor FRF_1

$$FRF_1 = 1 - .4(A + B)/L$$

d. Calculate the response frequency f_n

$$f_n = (FRF_1)(f_{ne})$$

6. Frequency ratio criterion

Calculate

$$r = f_s/f_n$$

The quotient r must not exceed 0.80

7. Bending stress criterion

a. Calculate the dynamic intensification factor K

$$K = 1/(1-r^2)$$

b. Calculate the fluid coefficient C

$$C = \sqrt{1.44 + 1.69K^2} \quad \text{for } N_R < 10^5$$

$$C = \sqrt{9 + K^2}/4 \quad \text{for } N_R > 10^5$$

c. Calculate shielding parameter α (cf. Figure D-1)

$$\alpha = SL/L$$

d. Calculate tapering parameter p^*

$$p^* = (A - B)/A$$

(The asterisk here is to avoid confusion with pressure p)

e. Calculate $\bar{\alpha} = (2 - p^* - \sqrt{4p^*-3p^{*2}})/2p^*$

f. Verify that $\alpha < \bar{\alpha}$ so that maximum bending stress is at the root. (Note: In practical cases it is unlikely that the maximum bending stress is other than at the root. The following steps assume $\alpha < \bar{\alpha}$. If $\alpha > \bar{\alpha}$, one must employ procedures and formulas developed in Appendix D.)

g. Calculate root stress intensity

$$\sigma^* = pd^2/(A^2-d^2) + .00018\ CU^2L^2(1-\alpha^2)(A+2B)A/v(A^4-d^4)$$

h. Root stress intensity criterion. It is required that

$$\sigma^* \leqq 1.5\ S_M$$

8. Fatigue life calculation

Calculate S_{alt}

$$S_{alt} = \begin{pmatrix} .00143 \\ .000275 \end{pmatrix} \frac{KU^2L^2(1-\alpha^2)A(A+2B)}{v(A^4-d^4)}$$

the upper figure being used for $N_R < 10^5$ and the lower for $N_R > 10^5$. It is required that $S_{alt} \leqq S_{end}$.

9. Attachment details verification

(No quantitative criteria are offered here. It is the definite recommendation of the writer that a part of the stress analysis of a thermowell also include the following steps.)

a. Assure that the attachment weld is a full penetration weld for which a definite and approved "welding procedure" instruction and inspection instructions are properly promulgated.

b. Assure that the "branch connection" rules of the applicable code are satisfied.

10. Remarks

a. Many steps above may be omitted in actual routine calculation. For example, usually it is not necessary to calculate p* (step 7d) or $\bar{\alpha}$ (step 7e) since α is usually less than 1/3 and the maximum stress will occur at the root regardless of the value of the tapering parameter p*.

b. The criteria above have been developed so as to assure reliable performance without economic penalty. Failure to satisfy any criterion does not necessarily imply unsatisfactory performance. In many cases sound reasons may be developed to permit violating one or more of these criteria. Such reasons may be developed on the basis of the analysis and discussions contained in this report. However, it may be costlier to develop such reasons than to strengthen the design.

11. Bibliography

The publications listed below may be obtained from the ASME (American Society of Mechanical Engineers) Publication Department, United Engineering Center, 345 East 47th Street, New York, N. Y., 10017.

(F1) ASME Boiler and Pressure Vessel Code Case No. 1331 (latest edition)

(F2) ASME Boiler and Pressure Vessel Code, Section III Nuclear Power Plant Components, Appendix 1.

(F3) ASME Steam Tables, 1967

(F4) ASME Boiler and Pressure Vessel Code, Section VIII, Division 1, Unfired Pressure Vessels.

(F5) USA Standard Code for Pressure Piping, USAS B31.1.0-1967 Power Piping

APPENDIX G

NUMERICAL EXAMPLE OF CALCULATION

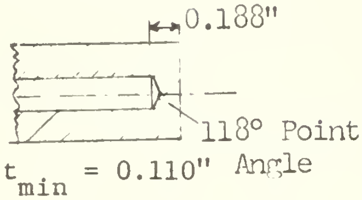
1. Main steam service, XXX station, XXX owner, XXX location

$p = 2350$ psig, $T = 995^{\circ}F$, $U = 210$ feet per second

$A = 1.5''$, $B = 1.0''$, $d = 0.26''$, $t_{AVG} = .162''$, $L = 3.09''$, $SL = .375''$.

(Note: t_{AVG} obtained from figure at right)

$t_{AVG} = .188-(d \tan 31^{\circ})/6 = .162''$



Material matches P22. Thus

$S_M = (11000)(.1) + (7800)(.9) = 8120$ psi

$S_{end} = (2S_a)_{1000000} = (2)(9000)$ appr. $= 18000$ psi

$E = [(23)(.95) + (24.5)(.05)] \times 10^6 = 23.1 \times 10^6$ psi

$\gamma = .283$ pounds per cubic inch (sufficiently accurate).

Absolute pressure = 2365 psia. Use double interpolation to get v.

	2300	2400	
$1000^{\circ}F$	.3372	.3214	$v = .3252$
$990^{\circ}F$	.3336	.3179	

(From pages 182 and 184 of ASME Steam Tables)

Kinematic viscosity $\nu = .0064$ (ft^2/1000 sec)

(From page 295 of ASME Steam Tables)

2. $d/B = .26$; $P_a = 7571$; $p = 2350 < P_a = 7571$ (OK)

3. $d\sqrt{p/S_M} = .140''$; $t_{AVG} = .162 > .140''$; $t_{min} = .110 > .070''$ (OK)

4. $N_R = (83)(210)(1.5)/(.0064) = 4.1 \times 10^6$; $N_S = .45$

$f_s = (12)(.45)(210)/(1.5) = 756$ Hz

5. $d/A = .173$, $B/A = .667$, $F_f = 3.31$ from Figure C-5,

[or more conservatively $F_f = 1.65 + (1.21)(1.5)(1-.094 \times 1.5) = 3.21$;

the alternate conservative formula gives $F_f = 2.70 + .54 = 3.24$].

$\gamma + \gamma' = .283 + (.00058)/(.3252) = .285$

$f_{ne} = [(3.31)(1.5)/(3.09)^2] \sqrt{(23.1\times10^6)/(.285)} = 4682$ Hz

$FRF_1 = 1-.4(2.5)/3.09 = .6764$; $f_n = 3167$ Hz

6. $r = 756/3167 = .239$. This is less than 0.8 (OK)

7. $K = 1.061$; $C = \sqrt{9 + (1.061)^2}/4 = .796$; $\alpha = .121 < .133$. Thus maximum <u>is</u> at the root. (We also verify this by calculating $p^* = .333$, $\bar{\alpha} = 1.0 > \alpha$)

$$\sigma^* = \frac{(2350)(.26)^2}{(1.5)^2-(.26)^2} + \frac{(.00018)(.796)(210)^2(3.09)^2(.985)(3.5)(1.5)}{(.3252)[(1.5)^4-(.26)^4]}$$

$$= 72.8 + 189.7 = 262.5 < (1.5)(8120) = 12180 \quad \text{(OK)}$$

8. $$S_{alt} = \frac{(.000275)(1.061)(210)^2(3.09)^2(.985)(1.5)(3.5)}{.3252[(1.5)^4-(.26)^4]} = 386 < 18000 \quad \text{(OK)}$$

In the preceding calculations all criteria were satisfied by very comfortable margins. However, if the thermowell were longer, say L = 6.50 inches, this would not be the case. We would calculate $f_{ne} = (4682)(3.09/6.50)^2 = 1058$ Hz; $FRF_1 = .846$; $f_n = 895$; Hz; $r = .845$. This violates the criterion $r \leqq .80$. Looking at all the other criteria we note quickly that they continue to be satisfied. We recall that with $N_R = 4.1 \times 10^6$ we are in the regime where N_s may become as large as .45 but that vortex shedding is not coherent. Accordingly a good argument could be advanced for permitting the value $r = .845$ rather than limiting it to .80. However, it might be more expedient to consider using a somewhat shorter thermowell, say L = 6.0 inches, for which all criteria would be satisfied.

There is one additional important step, cf., Section 9 of the preceding Appendix F.

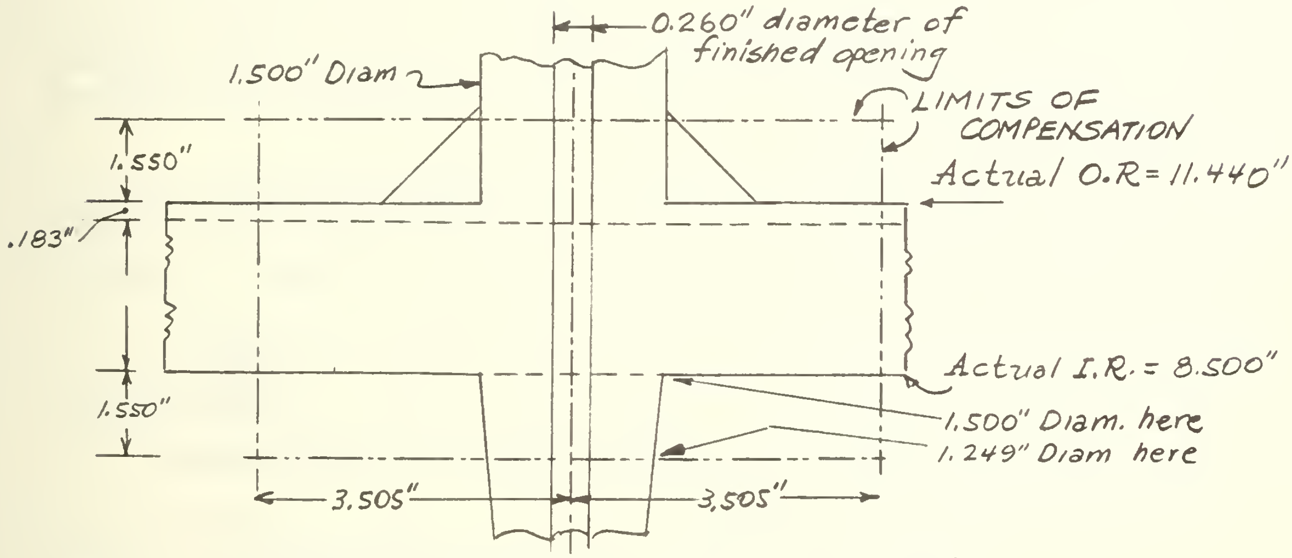
We will presume here that the standing instructions to fabricators and inspection personnel call for an appropriate full penetration weld with an appropriate welding procedure instruction. The service in this example is main steam service and we now must investigate compliance with

PG-32, et seq. of Section I (Power Boilers) of the ASME Boiler and Pressure Vessel Code. We assume that the MS piping specification calls for 17 inch I.D. × 2.938 minimum wall thickness of SA 213 T22. At 995F the S-value (S_M) is 8120 psi and the y-value (ferritec material) is .63. The formula of Par. PG27.2.2 may be written in terms of internal diameter d as

$$t = Pd/[2S-2P(1-y)]$$

$$= (2350)(17)/(2)(8120-2350 \times .37) = 2.755 \text{ in.}$$

Thus we can regard the pipe as having O.D. = 22.510 inches with W.T. = 2.755 in., and with an excess thickness of 2.938 - 2.755 = .183 inches available for reinforcement. As a check, we note that (2350)(22.510)/(2)(8120 + .63 × 2350) = 2.755 in.



(Details of shielding not shown. See remark at end of this Appendix.)

Figure G-1 Reinforcement calculation diagram (N.T.S.)

The limits of compensation measured parallel to the vessel (pipe) wall are (Par. PG-36.2.2) .130" + 2.755" + 1/2(1.5-.26)" = 3.505" on each side of the centerline, and, (Par. PG-36.3.2), (2 1/2)(.620) + .183 = 1.733 externally and 1.550 internally. The total area of compensation required (Par. PG-33.2) is A = (.260) × (2.755) × (1) = .716 sq. in.

The compensation provided consists of

$A_1 = (7.010-.260)(.183) = 1.235$ sq. in., excess wall thickness

$A_2 = (1.500-.260)(1.550) = 1.922$ sq. in., nozzle, external

$A_3 = (.5 \times 2.749-.260)(1.550) = 1.727$ sq. in., nozzle, internal

$A_4 =$ (not necessary) = fillets, within comp. limits

The total, not counting applicable area of fillet welds, is 4.884 sq. in. which greatly exceeds the requirement of 0.716 sq. in. Accordingly, the reinforcement rules are much more than adequately satisfied. It is expected that this will always prove to be the case for high-pressure, high-temperature installations but it may not always automatically be so for less severe service, for example, for cold reheat service. In any case, however, a procedure for assuring compliance with the applicable reinforcement rules is a definite part of the recommended criteria and procedure in this report.

(Note: this example is intended to illustrate a general type of calculation, and thus includes a "shielded length" SL = 0.375". However, no indication is made of the detail which accomplishes the shielding. In particular, no such detail is shown in Fig. G-1, and it is presumed that the detail, whatever it is, does not enter into the calculation of branch connection compensation.)

INITIAL DISTRIBUTION LIST

Mrs. J. E. Leary Chief Control and Instrumentation Engineer Bechtel Power Corporation P.O. Box 3965 San Francisco, CA 94119	2
Mr. F. E. Vinson Bechtel Power Corporation P.O. Box 3965 San Francisco, CA 94119	1
Mr. S. E. Moore Oak Ridge National Laboratory P.O. Box Y Oak Ridge, Tennessee 37830	1
Mr. L. A. Dodge 364 Dunbarton Boulevard Richmond Heights, Ohio 44143	1
Mr. C. Barnhardt Bechtel Power Corporation P.O. Box 3965 San Francisco, CA 94119	1
Mr. W. O. Hayes, Secretary Secretary ASME Committee PB51 ASME 345 East 47th Street New York, NY 10017	20
Mr. W. N. Wright TemTex Systems 5619 Dyer Street Dallas, Texas 75206	1
Mr. John Archer Gilbert Associates, Inc. Reading, Pennsylvania 19603	1
Mr. A. Lohmeier SGDE; ASME B&PV Committee Westinghouse Electric Corporation Power Systems Box 19218 Tampa, Florida 33616	1
Mr. R. F. Abrahamsen ASME 345 East 47th Street New York, NY 10017	1

Mr. A. Partington
Westinghouse Electric Corporation
Y-4 Technical Development
Lester, Pennsylvania 19113 1

Mr. J. W. Murdock
Consulting Engineer
684 Old School House Drive
Springfield, Pennsylvania 19064 1

Mr. Norman Blair
Chairman B31 MCD
4836 N. Earle Street
Rosemead, California 91770 1

Mr. D. M. Fischer
Secretary B31 MDC
Mail Code 4307
Amoco Chemicals Corporation
P.O. Box 8640A
Chicago, Illinois 60680 14

Professor J. E. Brock
Code 59BC
Department of Mechanical Engineering
Naval Postgraduate School
Monterey, California 93940 5

Department of Mechanical Engineering
Code 59
Naval Postgraduate School
Monterey, California 93940 1

Library
Naval Postgraduate School
Monterey, California 93940 2

Dean of Research Administration
Naval Postgraduate School
Monterey, California 93940 2

Director
Defense Documentation Center
5010 Duke Street
Alexandria, Virginia 22314 2